Bowling ball dynamics a la Hopkins and Patterson (1977)

Andrew Kickertz

Outline

- Introduction to bowling parameters
- Equations of motion
- Simulation with Matlab
- Results
- Assumptions and real life

Ball properties

- 27” circumference
- = 0.109 m radius
- 6 – 16 lbs

Hopkins and Patterson (1977)

Ball properties

- 27” circumference
- = 0.109 m radius
- 6 – 16 lbs

- All radii of gyration 2.46 – 2.80”
- Solid sphere = 2.78”
- Max differential 0.06”

Lane properties

- 41” wide
- 60’ long

Hopkins and Patterson (1977)

Assumptions

- Uniform sphere
- Coulomb friction
- Uniform friction

Hopkins and Patterson (1977)

Setup

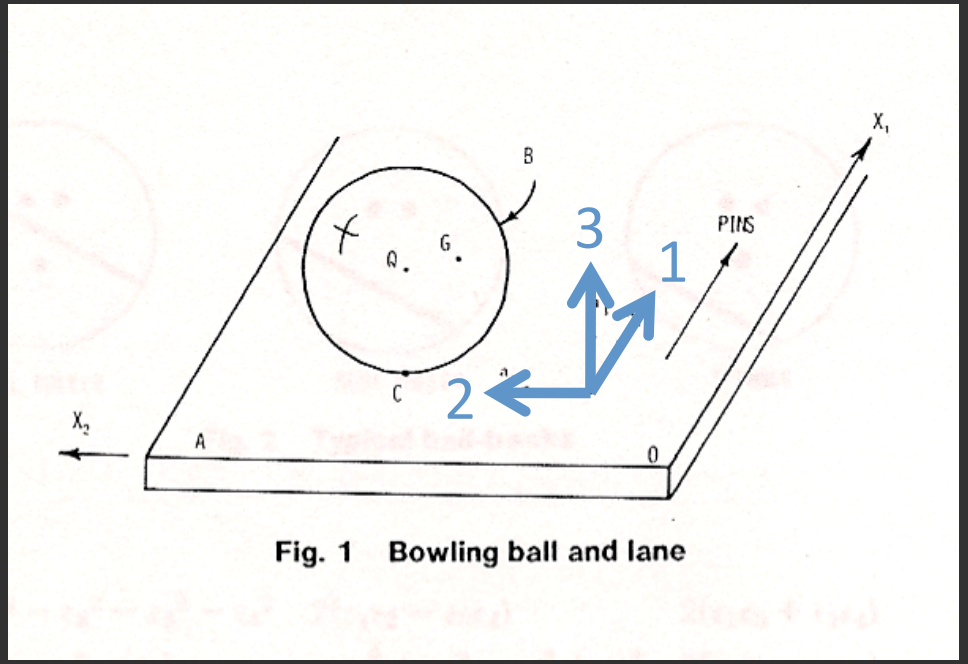


Fig. 1 Bowling ball and lane

Hopkins and Patterson (1977)

Setup

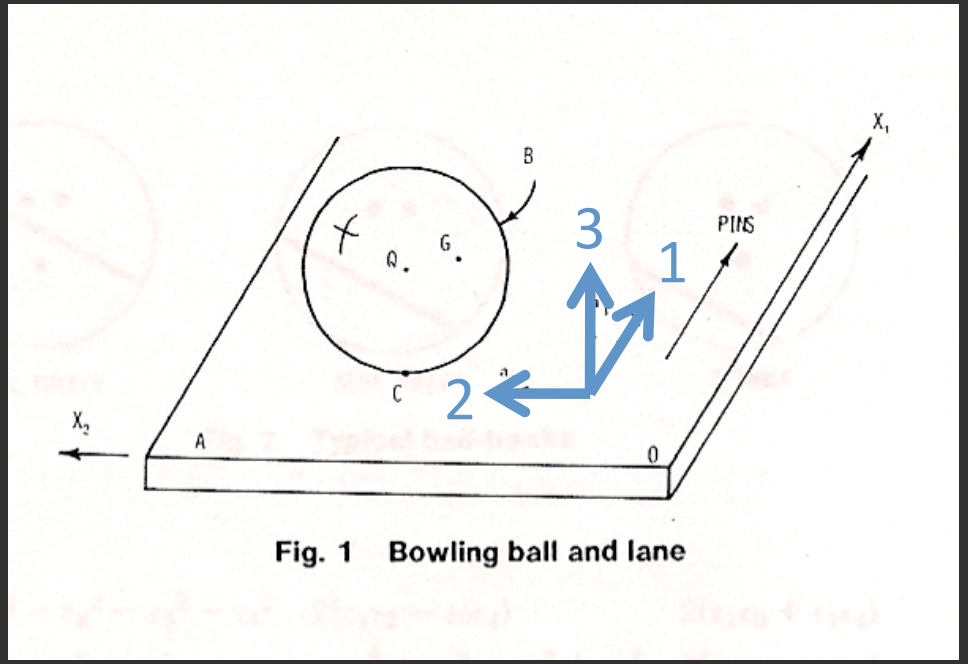
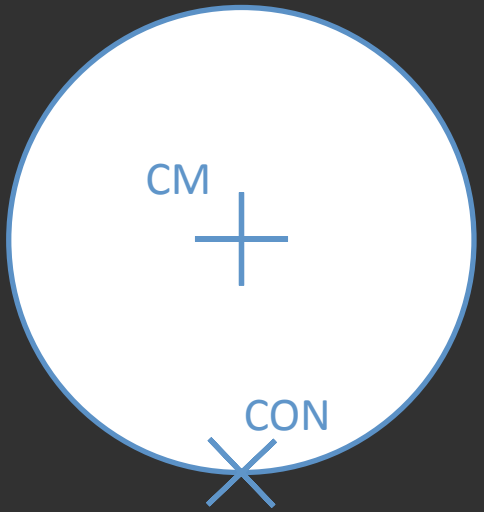


Fig. 1 Bowling ball and lane



Hopkins and Patterson (1977)

Equations of motion

$$\mathbf{v}_{con} = \mathbf{v}_{cm} + \omega \times -r\hat{\mathbf{k}}$$

$$\mathbf{F} = -\mu m g \frac{\mathbf{v}_{con}}{|\mathbf{v}_{con}|}$$

$$\dot{\mathbf{v}}_{cm} = \frac{\mathbf{F}}{m}$$

Hopkins and Patterson (1977)

Equations of motion

$$\begin{aligned} \mathbf{v}_{con} &= \mathbf{v}_{cm} + \omega \times -r\hat{\mathbf{k}} \\ \mathbf{F} &= -\mu m g \frac{\mathbf{v}_{con}}{|\mathbf{v}_{con}|} \\ \dot{\mathbf{v}}_{cm} &= \frac{\mathbf{F}}{m} \end{aligned}$$

$$\begin{aligned} \tau &= -r\hat{\mathbf{k}} \times \mathbf{F} \\ \dot{\omega} &= \frac{\tau}{I} \end{aligned}$$

Hopkins and Patterson (1977)

Code

$y(1) = v_x$

$y(2) = v_y$

$y(3) = w_x$

$y(4) = w_y$

$y(5) = x$

$y(6) = y$

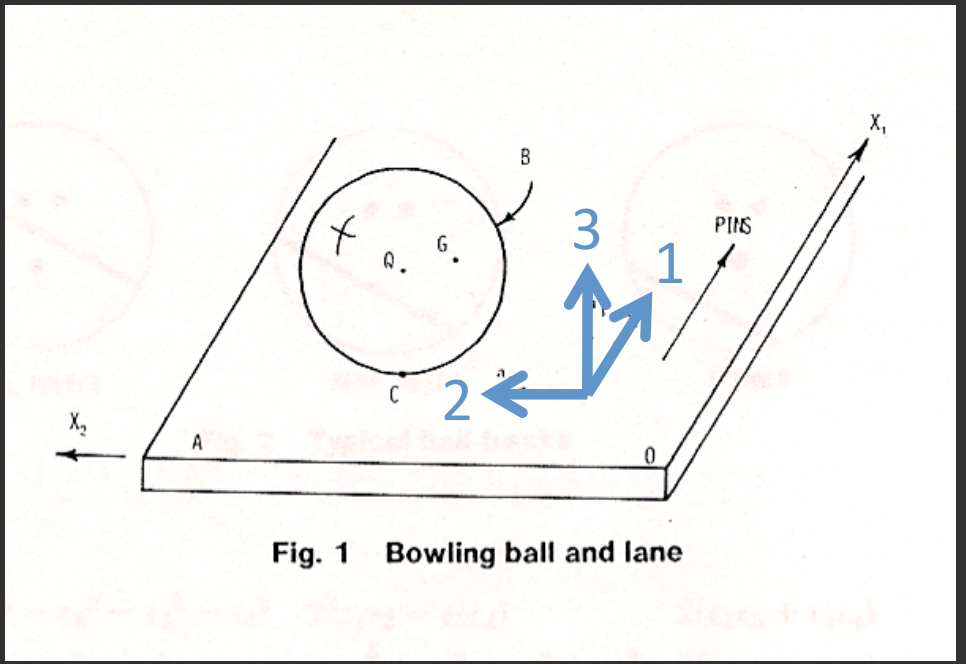


Fig. 1 Bowling ball and lane

Code

```
function dy = ode1(t,y)
r  = 0.107899;
m  = 7;
I  = 2/5*m*r^2;
mu = 0.09;
g  = 9.8054;
```

Code

```
function dy = ode1(t,y)
r  = 0.107899;
m  = 7;
I  = 2/5*m*r^2;
mu = 0.09;
g  = 9.8054;
```

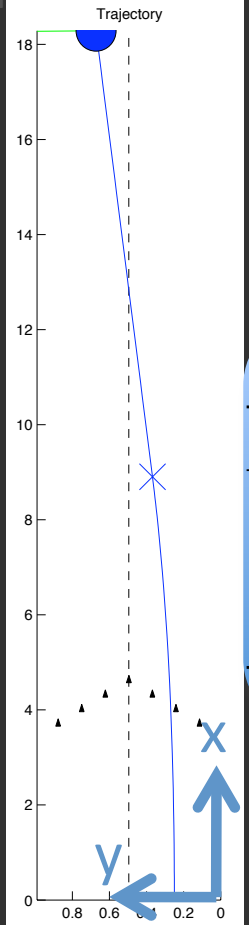
```
vcm  = [y(1) y(2) 0];
w    = [y(3) y(4) 0];
vcon = vcm + cross(w,[0 0 -r]);
F    = -mu*m*g*vcon/
          norm(vcon);
tau  = cross([0 0 -r],F);
```

Code

```
dy = zeros(6,1);
dy(1) = dot(F/m,  [1 0 0]);
dy(2) = dot(F/m,  [0 1 0]);
dy(3) = dot(tau/I,[1 0 0]);
```

```
dy(4) = dot(tau/I,[0 1 0]);
dy(5) = y(1);
dy(6) = y(2);
```

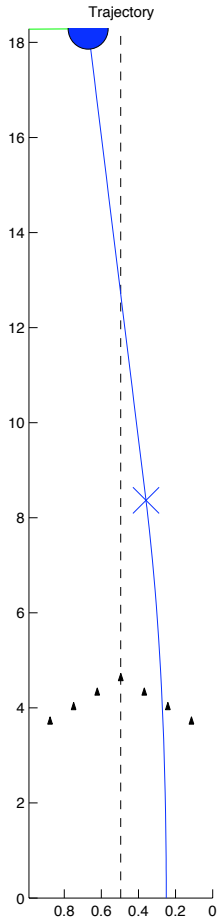
Demo

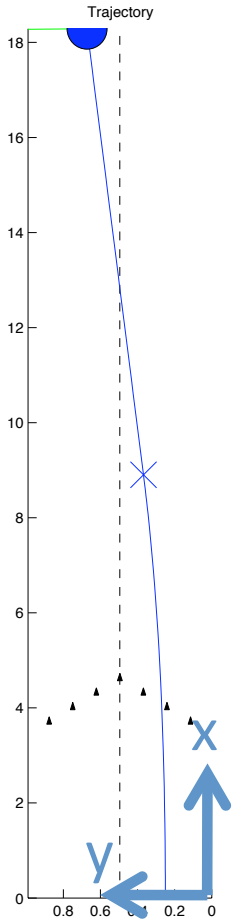


Initial conditions

Table 1: Hopkins and Patterson cases

Case	vox (ft/s)	voy (ft/s)	wox (rad/s)	woy (rad/s)
1	18	0	-4	0
2	18	0	-4	-4
3	18	0.5	-4	-4
4	24	0.5	-4	-4

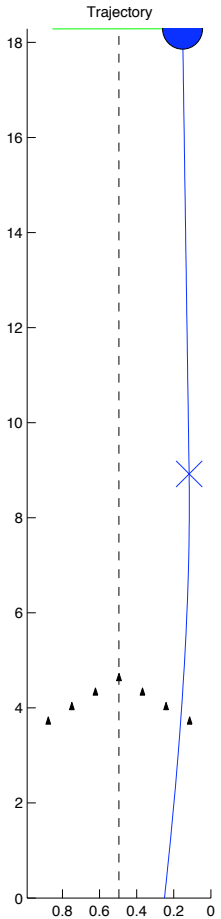


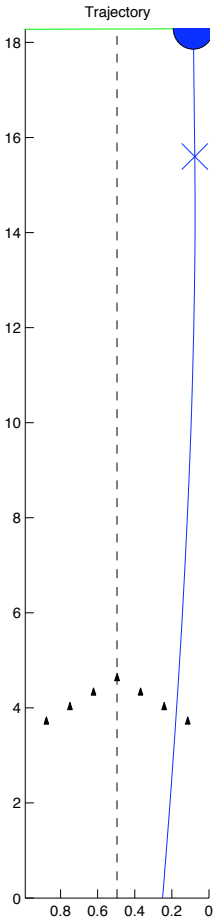


Trajectories

Table 1: Hopkins and Patterson cases

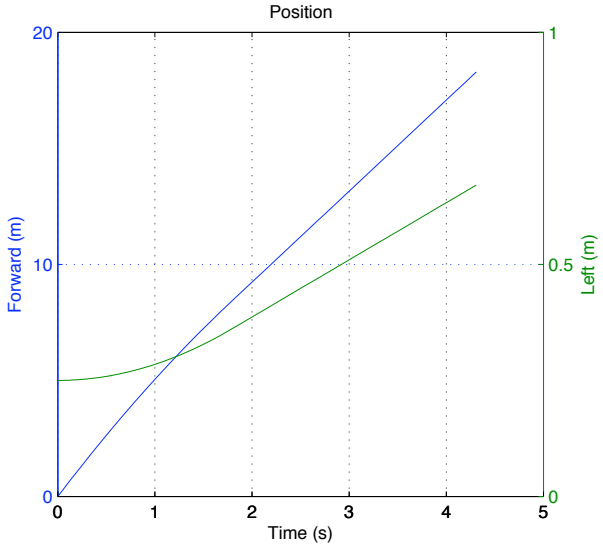
Case	vox (ft/s)	voy (ft/s)	wox (rad/s)	woy (rad/s)
1	18	0	-4	0
2	18	0	-4	-4
3	18	0.5	-4	-4
4	24	0.5	-4	-4

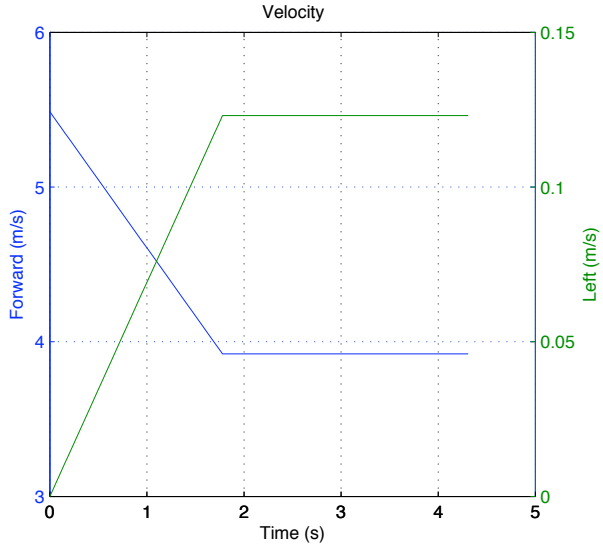


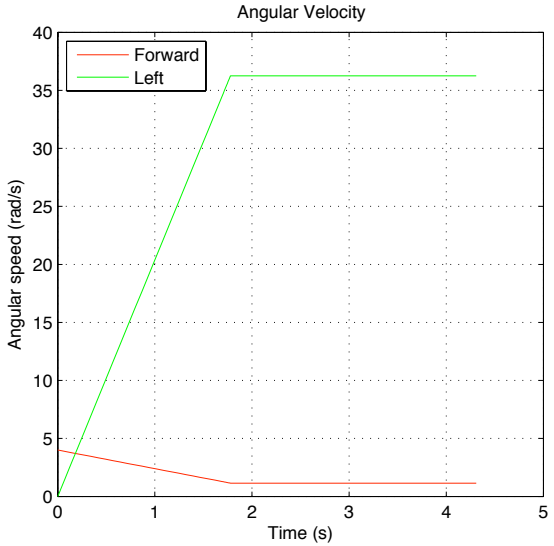


Hopkins and Patterson (1977)

Kinematics







Hopkins and Patterson (1977)

Conclusions

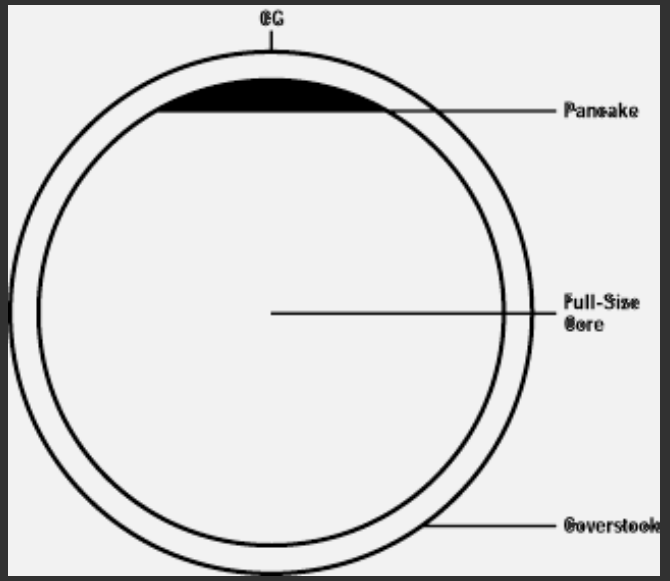
- Analytic solution possible
- Parabolic path
- Entry angle independent of μ

Assumptions

- ~~Uniform sphere~~
- Coulomb friction
- Uniform friction

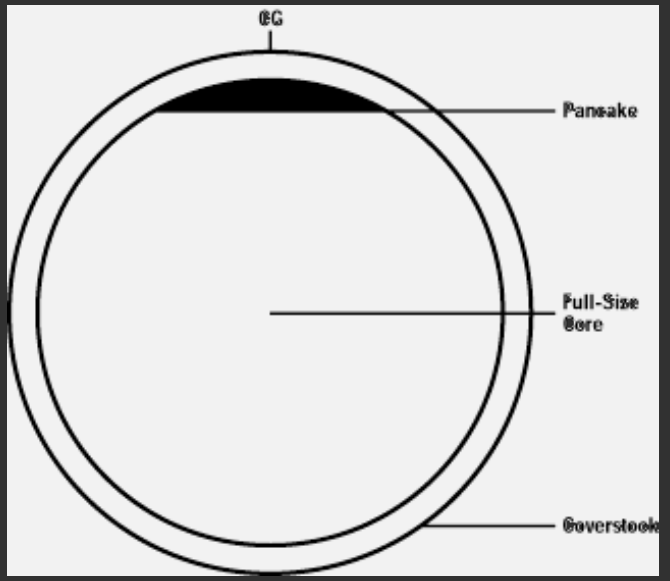
Hopkins and Patterson (1977)

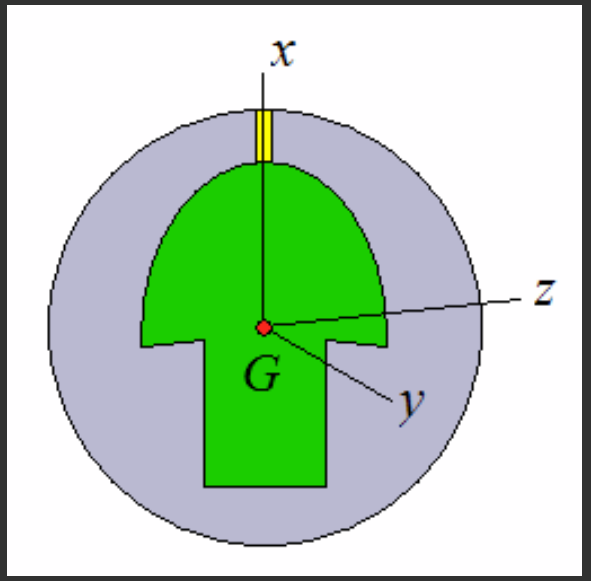
Uniform sphere?



Hopkins and Patterson (1977)

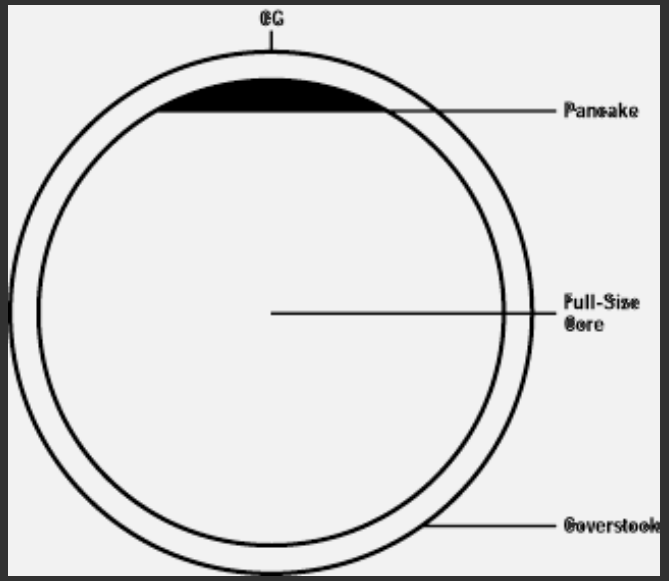
Uniform sphere?

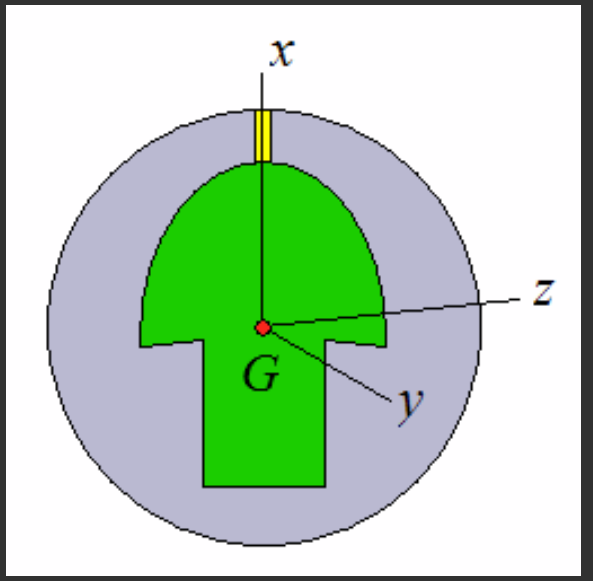


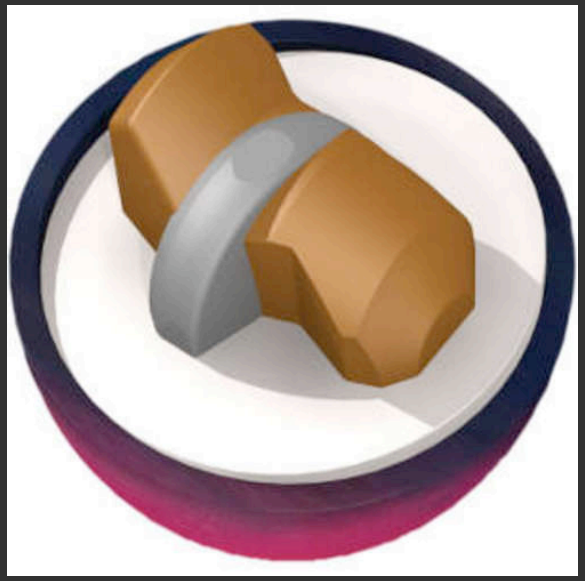


Hopkins and Patterson (1977)

Uniform sphere?







Hopkins and Patterson (1977)

Dealing with reality

- Mass center offset
- Inertia matrix
- Euler parameters

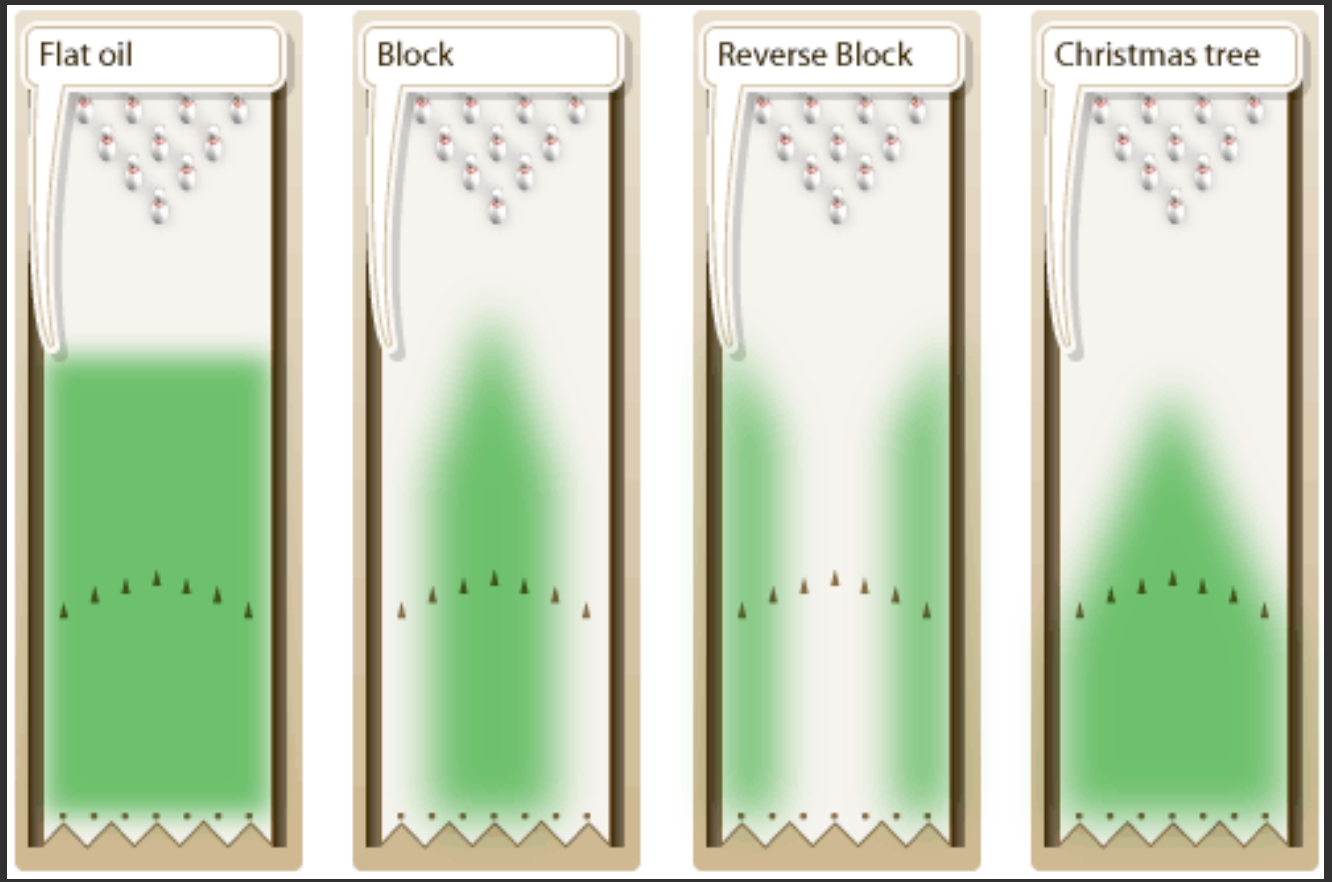
Hopkins and Patterson (1977)

Assumptions

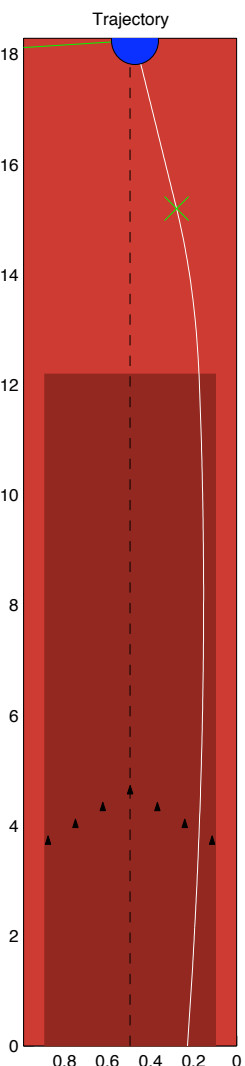
- ~~Uniform sphere~~
- Coulomb friction - OK
- Uniform friction

Assumptions

- ~~Uniform sphere~~
- Coulomb friction - OK
- ~~Uniform friction~~



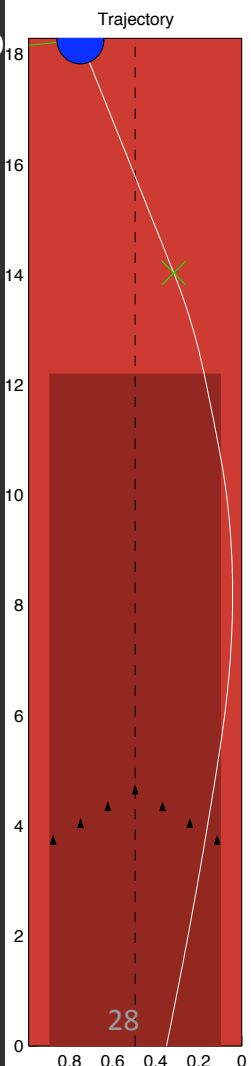
Hopkins and Patterson (1977)

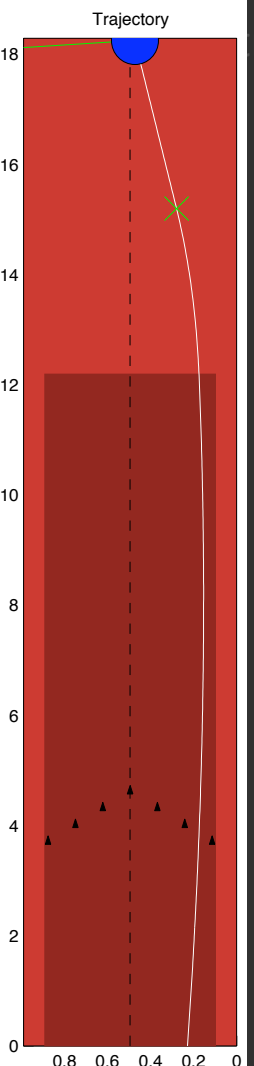


Block oil pattern

- Can use logic
- Or a matrix of μ's

Hopkins and Patterson (1977)





Block oil pattern

```
if(q2<0.09906 || q2>0.89154 || q1>12.192)
    out = 0.2;
else
    out = 0.04;
end
```

The end