Bowling Ball Dynamics

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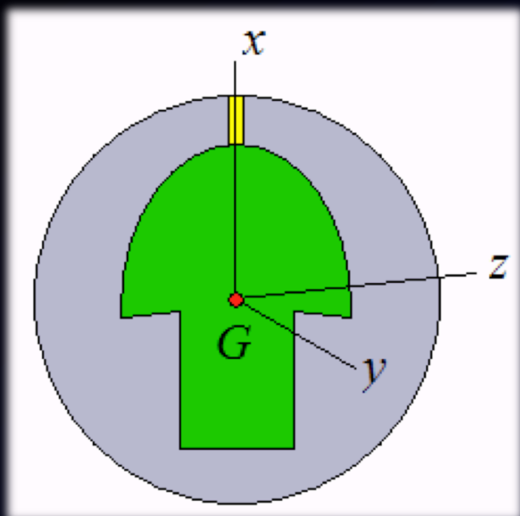
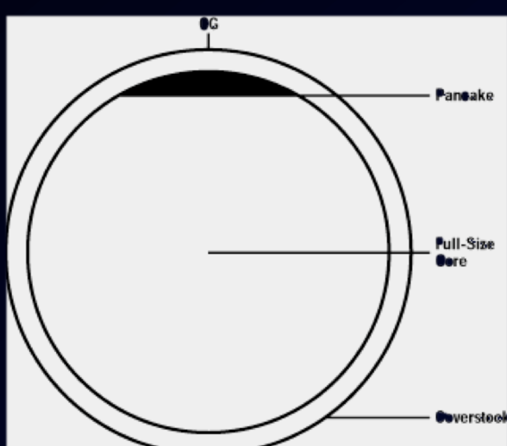
Ball Properties

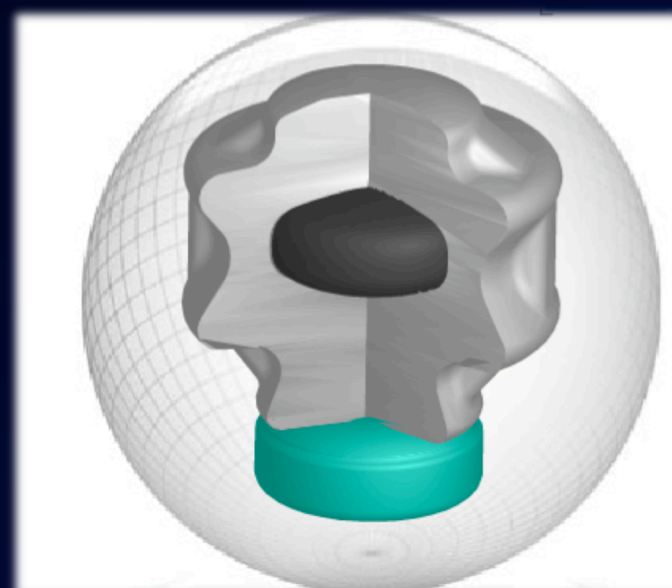
- 27” circumference -> 8.59” diameter
- 6 – 16 lbs.
- All radii of gyration must be 2.43 – 2.80”
  – Solid sphere would be 2.78”
- Maximum differential 0.080”

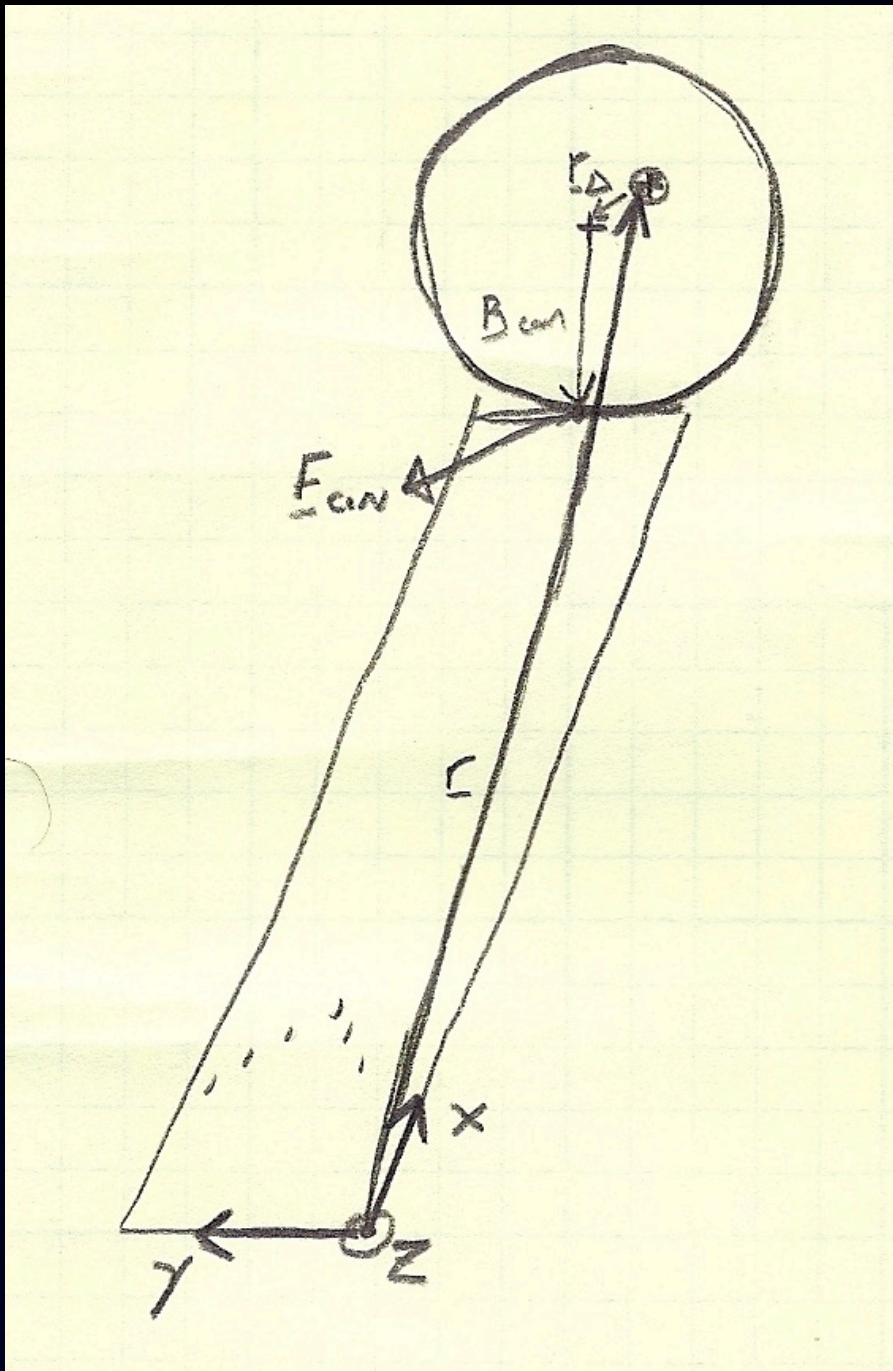
http://www.youtube.com/watch?v=lS-0f0v-fAA

Example Geometries









What makes bowling balls hook?

Equations of Motion

$$M\ddot{\vec{r}} = \vec{F}_{\text{con}} + \vec{F}_{g}\,, \tag{1}$$

$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = (\vec{r}_{\Delta} + \vec{R}_{\text{con}}) \times \vec{F}_{\text{con}}\,, \tag{2}$$

$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = (\vec{r}_\Delta + \vec{R}_{\text{con}}) \times \vec{F}_{\text{con}},$$

Expand Left Side

$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = \mathbf{I}\vec{\alpha} + \vec{\omega} \times (\mathbf{I}\vec{\omega}), \tag{4}$$

$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = (\mathbf{I}_o + \mathbf{I}_{\text{dev}})\vec{\alpha} + \vec{\omega} \times (\mathbf{I}_{\text{dev}}\vec{\omega}). \tag{5}$$

$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = (\vec{r}_\Delta + \vec{R}_{\mathrm{con}}) \times \vec{F}_{\mathrm{con}},$$

Expand Right Side: SLIDING

$$\vec{F}_{\mathrm{con}} = F_n(\mu_x, \mu_y, 1) = F_n\vec{\mu}. \tag{A3}$$

$$F_n = M(g - \ddot{r}_{\Delta,z}), \tag{A4}$$

$$\ddot{\vec{r}}_\Delta = \frac{d\dot{\vec{r}}_\Delta}{dt} = \frac{d(\vec{\omega}\times\vec{r}_\Delta)}{dt} = \vec{\alpha}\times\vec{r}_\Delta + \vec{\omega}\times(\vec{\omega}\times\vec{r}_\Delta). \tag{A5}$$

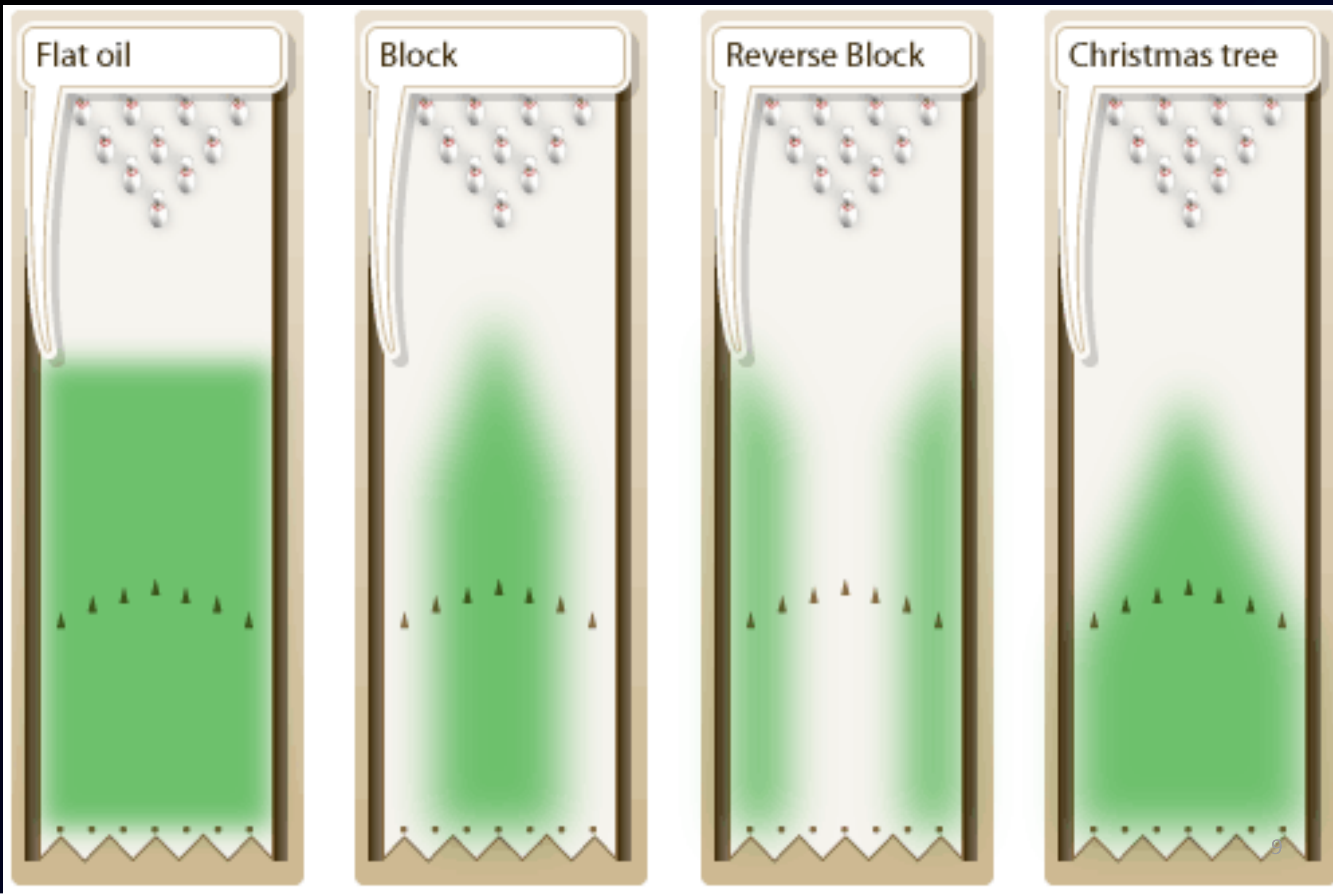
$$F_n = M(g + a_\alpha + a_\omega), \tag{A6}$$

where

$$a_\alpha = [\vec{r}_\Delta \times \vec{\alpha}]_z \quad \text{and} \quad a_\omega = [(\vec{\omega}\times\vec{r}_\Delta)\times\vec{\omega}]_z, \tag{A7}$$

$$(\mathbf{I}_o + \mathbf{I}_{\mathrm{dev}})\vec{\alpha} + \vec{\omega}\times(\mathbf{I}_{\mathrm{dev}}\,\vec{\omega}) = (\vec{r}_\Delta + \vec{R}_{\mathrm{con}})\times(g + a_\alpha + a_\omega)\vec{\mu}. \tag{A8}$$

Oil Patterns



$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = (\vec{r}_\Delta + \vec{R}_{\text{con}}) \times \vec{F}_{\text{con}},$$

Expand Right Side: SLIDING (2)

$$= \underline{\tau}_{dev}$$

$$(\underline{\underline{I}}_0 + \underline{\underline{I}}_{dev})\underline{\alpha} + \underline{\omega} \times (\underline{\underline{I}}_{dev}\underline{\omega}) = (\underline{r}_\Delta + \underline{R}_{con}) \times (g + a_\alpha + a_\omega)\,\underline{\mu}$$

$$= g\left[(\underline{r}_\Delta + \underline{R}_{con}) \times \underline{\mu}\right] + a_\alpha\left[(r_\Delta + \underline{R}_{con}) \times \underline{\mu}\right] + a_\omega\left[(\underline{r}_\Delta + \underline{R}_{con}) \times \underline{\mu}\right]$$

$= \underline{\tau}_{fric}$ ish

$$= -\underline{\underline{I}}^s_\Delta\,\underline{\alpha} - \underline{\underline{I}}^s_{\Delta\Delta}\,\underline{\alpha}$$

$= \underline{\tau}^s_{\Delta\Delta} + \underline{\tau}^s_\Delta$ ish

$$\underline{\underline{I}}^s_\Delta = R_{ball}\left[\qquad\right] \qquad \underline{\underline{I}}^s_{\Delta\Delta} = \left[\qquad\right]$$

$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = (\vec{r}_\Delta + \vec{R}_{\text{con}}) \times \vec{F}_{\text{con}},$$

Expand Right Side: SLIDING (3)

$$\mathbf{I}_\Delta^s = R_{\text{ball}} \begin{bmatrix} r_{\Delta y}\mu_y & -r_{\Delta x}\mu_y & 0 \\ r_{\Delta y}\mu_x & r_{\Delta x}\mu_x & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{A9}$$

then $\mathbf{I}_\Delta^s \vec{\alpha} = -a_\alpha \vec{R}_{\text{con}} \times \vec{\mu}$. And if

$$\mathbf{I}_{\Delta\Delta}^s = \begin{bmatrix} r_{\Delta y}(r_{\Delta y} - r_{\Delta z}\mu_y) & -r_{\Delta x}(r_{\Delta y} - r_{\Delta z}\mu_y) & 0 \\ r_{\Delta y}(r_{\Delta z}\mu_x - r_{\Delta x}) & -r_{\Delta x}(r_{\Delta z}\mu_x - r_{\Delta x}) & 0 \\ r_{\Delta y}(r_{\Delta x}\mu_y - r_{\Delta y}\mu_x) & -r_{\Delta x}(r_{\Delta x}\mu_y - r_{\Delta y}\mu_x) & 0 \end{bmatrix}, \tag{A10}$$

then $\mathbf{I}_{\Delta\Delta}^s \vec{\alpha} = -a_\alpha \vec{r}_\Delta \times \vec{\mu}$. For the remaining terms we can define various torques:

$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = (\vec{r}_\Delta + \vec{R}_{\text{con}}) \times \vec{F}_{\text{con}},$$

Expand Right Side: SLIDING (4)

$$\vec{\tau}_{\text{fric}} = g\vec{R}_{\text{con}} \times \vec{\mu}, \quad \vec{\tau}_{\text{dev}} = (\mathbf{I}_{\text{dev}}\,\vec{\omega}) \times \vec{\omega},$$
$$\vec{\tau}^{\,s}_{\Delta} = g\vec{r}_{\Delta} \times \vec{\mu} + a_\omega \vec{R}_{\text{con}} \times \vec{\mu}, \quad \vec{\tau}^{\,s}_{\Delta\Delta} = a_\omega \vec{r}_\Delta \times \vec{\mu}. \tag{A11}$$

At last, alpha is only on the left, and everything on the right is written as a torque!

$$(\mathbf{I}_o + \mathbf{I}_{\text{dev}} + \mathbf{I}^s_{\Delta} + \mathbf{I}^s_{\Delta\Delta})\vec{\alpha} = \vec{\tau}_{\text{fric}} + \vec{\tau}_{\text{dev}} + \vec{\tau}^{\,s}_{\Delta} + \vec{\tau}^{\,s}_{\Delta\Delta}, \tag{A12}$$

$$\frac{d}{dt}(\mathbf{I}\vec{\omega}) = (\vec{r}_\Delta + \vec{R}_{\text{con}}) \times \vec{F}_{\text{con}},$$

Expand Right Side: ROLLING

$$(\mathbf{I}_o + \mathbf{I}_{\text{dev}} + \mathbf{I}_{\text{Roll}} + \mathbf{I}_\Delta^{r})\,\vec{\alpha} = \vec{\tau}_{\text{dev}} + \vec{\tau}_\Delta^{\,r} + \vec{\tau}_{\Delta\Delta}^{\,r}\,. \tag{B7}$$

Simulation

$\underline{r}, \dot{\underline{r}}, \ddot{\underline{r}}$

$\underline{\theta}, \dot{\underline{\theta}}, \ddot{\underline{\theta}}$

$\underline{r}_{\Delta}(\theta)$

$\underline{I}(\theta) \rightarrow \underline{I}_0 \ \& \ \underline{I}_{dev}$

Determine rolling v sliding from

$$\ddot{\vec{r}}_{\mathrm{cb}} = \vec{R}_{\mathrm{con}} \times \vec{\alpha}$$

Results

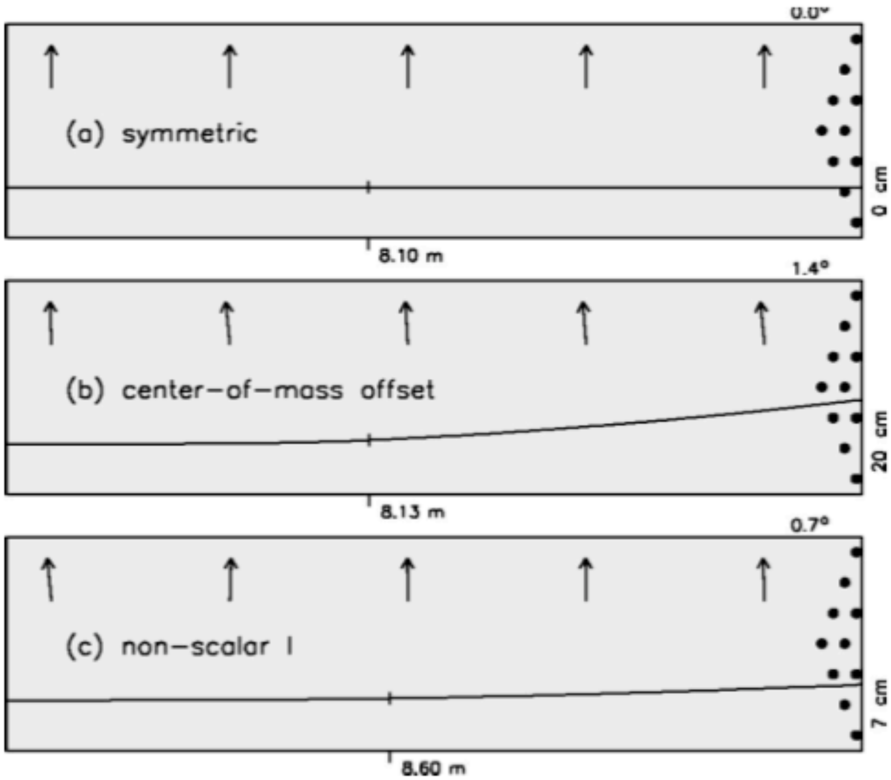
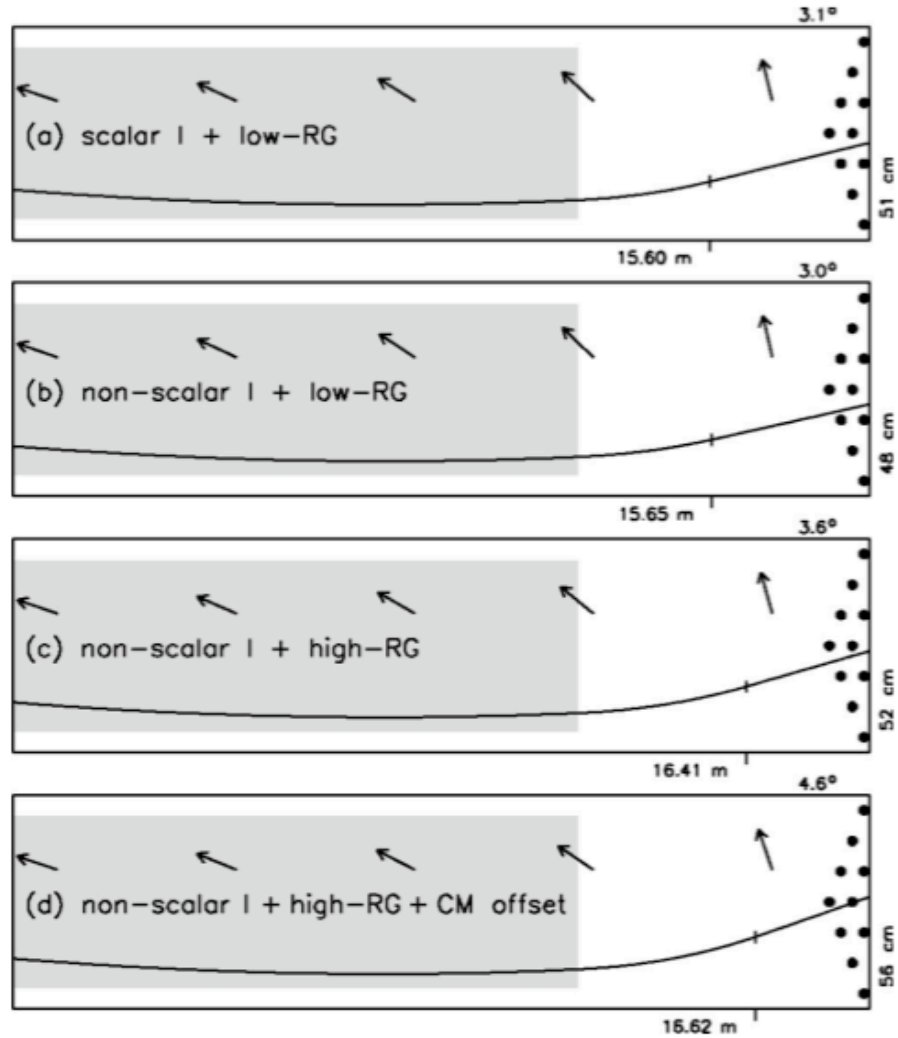


Fig. 5. Forward rotation. A ball delivered with forward rotation only will not hook if it is sufficiently symmetric [(a) case 1 in Table III]. But, in general, the ball will hook if there is either center-of-mass offset [(b) case 4 in Table III] or nonscalar moments of inertia [(c) case 5 in Table III]. Arrows indicate the direction of $\vec{\omega}$.



Simulations v Bowling Jargon

B1: High-RG balls get better length and good backend reaction.
P1: Increasing the moment of inertia (that is, the radius of gyration $\mathbf{R}_G$) makes a ball slide further and reach pins at a more oblique angle.

B2: Increasing the top or finger weight increases the length and backend reaction.
P2: If the center-of-mass offset r_Δ remains mostly on the left side of the ball during its trajectory, the ball slides further and reaches the pins at a more oblique angle.

B3: Balls with leverage drilling get more length and more backend reaction than balls with label or axis drilling.
P3: A ball drilled so that the initial rotation axis is midway between the principal rotation axes will slide further and reach pins at a more oblique angle.

Simulations v Bowling Jargon (2)

B4: Balls with high differential RG get more backend reaction.
P4: A ball where the radii of gyration are not all equal will reach the pins at a more oblique angle.

B5: Two balls may behave differently if their weight blocks have different shapes, even though they have identical coverstocks, positive weights, and RG.
P5: Balls with identical surface friction, center-of-mass offset, and moments of inertia may have different rotational properties.