Robustness and SMC

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Overview

- What is Robustness and why do we care?
- Different types of Robust Control Techniques
- Sliding Mode Control (SMC)
  - Definition and Benefits
  - Drawbacks and Requirements
- Applications of SMC
  - Inverted Pendulum
  - Aircrafts/Helicopters

Why We Should Care

- In the vietnam era, 20% of aircraft losses were due to flight control damage
  - Loss of hydraulics, actuator damage, and surface damage accounted for 80+%.
- Over 30% of todays aircraft would not flyable without advanced control systems.
- Control Failure Examples:
  - AA flight 96 DC10-1972 Explosive Decompression with severed flight controls to limit ailerons and elevator but no rudder. Still landed as a result of internal controls.
  - Japanese 747-1985 Faulty repair caused the tail and vertical stabilizer to be blown off. The pilots flew for another 32 minutes with limited control before crashing killing 520 people.
  - Philippines 747-1994 Hydraulics damaged by a bomb in passenger cabin. Landed 40 minutes later.
  - Baghdad-2003 Airbus A300 was first modern airliner to land with only engine controls.

Definition

Reconfigurable flight control is an automatic flight control system which is able to compensate for sudden, potentially large, unknown failure events in real time using online adaptive control laws guaranteeing system stability and achieving some level of required performance and handling qualities

Reconfigurable Flight Control

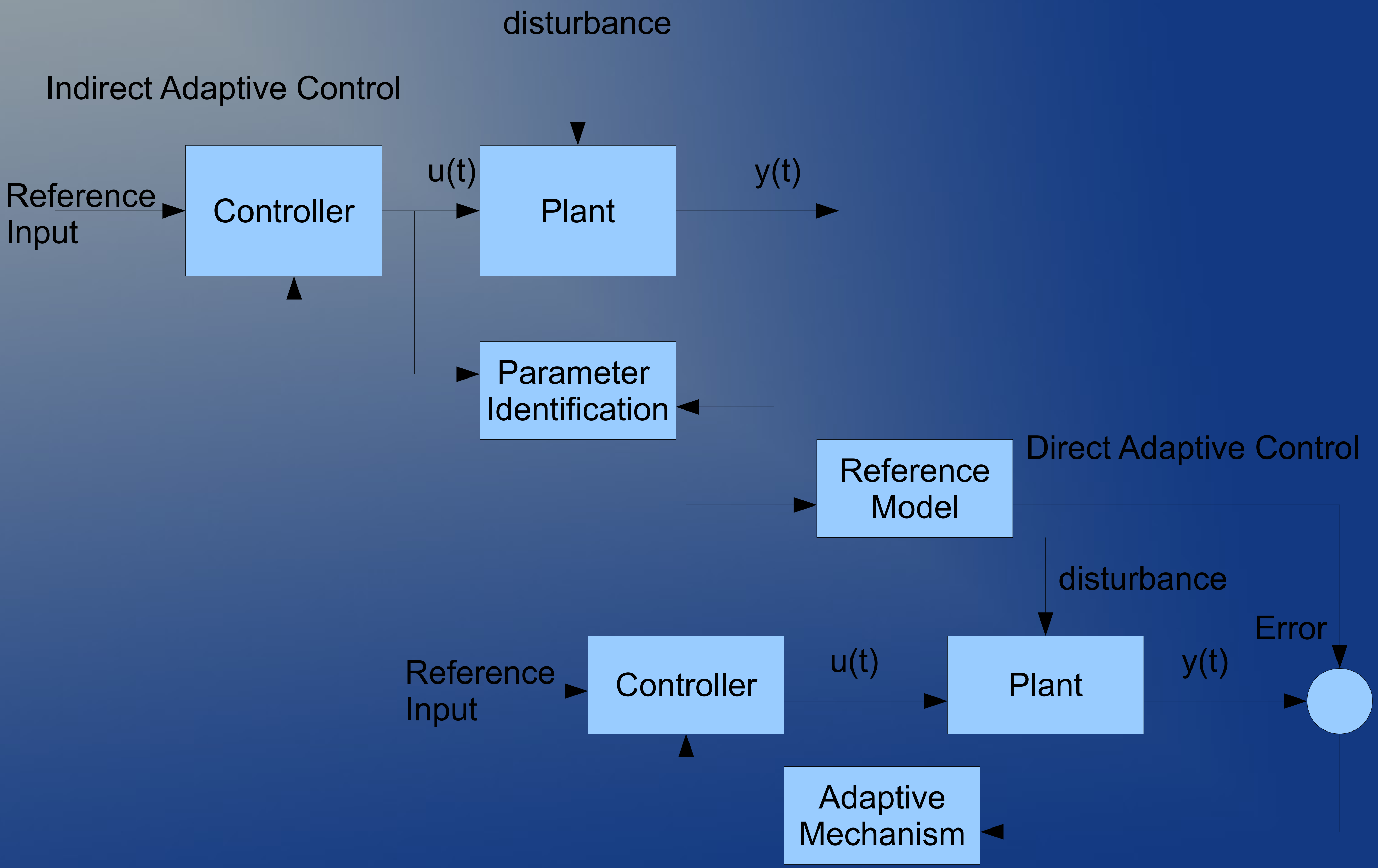
- Four main aspects to a flight control system
  - Failure Detection
  - System Parameter Identification
  - Flight control reconfiguration
  - Control allocation

- While Modern Control Systems officially starts in 1965, with the advent of small digital computers modern control design is centered around work mostly from the 80's-90's.

Adaptive Control Strategies

- Indirect Adaptive Control
  - Indirect control has the plant model constructed online by an observer/parameter control then an appropriate control law is calculated.
  - Indirect or explicit control has the benefit of separating the controller from the plant.
- Direct Adaptive Control
  - Synthesizes the controller utilizing performance criteria without explicit construction of a plant.
  - Direct or Implicit Control tends to be faster as there are less calls to the reference model.

Indirect vs Direct Adaptive Control



Indirect Control

- Two methods receive the most attention:
  - Receding Horizon Optimal Control (RHO)
  - Multiple Model Estimation (MMAE)
- RHO with least squares parameter ID or neural nets have been used on the ICE, F-16, MATV, and many unmanned vehicles.
- Other methods include:
  - Kalman filters
  - Model recasting
  - model reference adaptive controllers
  - Simple and modified PID

Direct Control

- Almost all direct methods include some form of model reference following or MRAC Systems.
- Model Reference Adaptive Control Systems have 4 parts:
  - The plant, which may be nonlinear, time-varying, and with unknown parameters
  - The reference model which is usually a lower order, linear, dynamic model which generates a desired closed loop system output response
  - A controller with time-varying components
  - Some type of adaptive algorithm which adjusts the controller based on the error.

Direct Control Options

- Some of the most popular methods are:
  - Dynamic Inversion: TAFA for the RESTORE
  - Backstepping
  - decentralized adaptive neuro-fuzzy designs
  - adaptive PI for the AFTI/F-16

- Perhaps the most notable attention is for the Sliding Mode Control method.

Sliding Mode Control

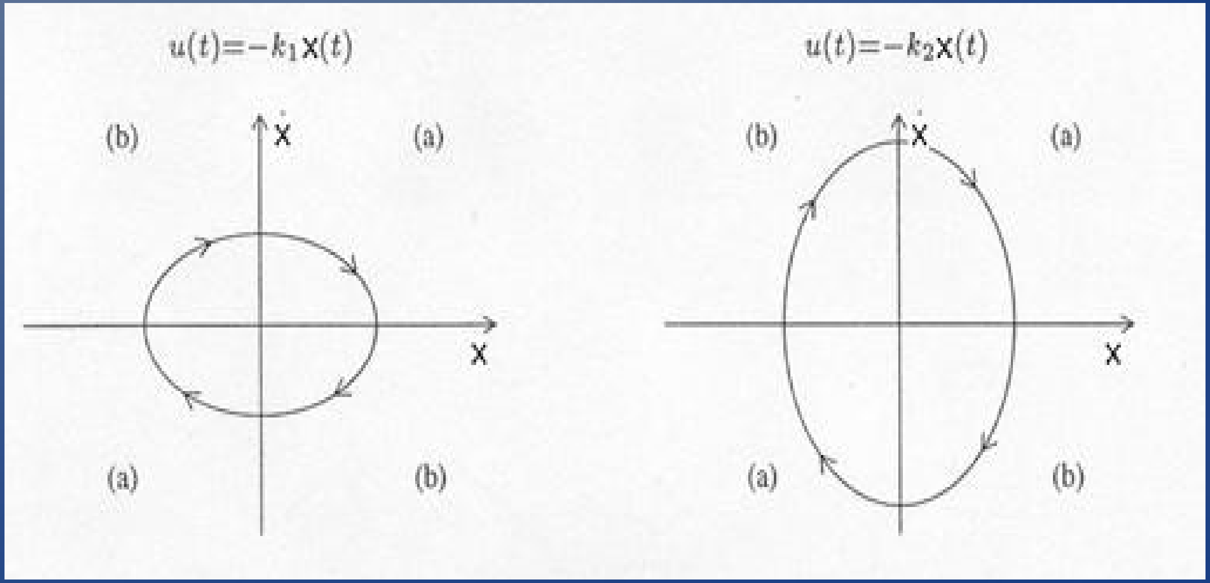
- SMC are a subset of controllers known as Variable Structure Controllers (VSC) which changes based on a predefined function of the states of the system.
- Applications of the SMC include:
    - Robotic control, motor control, flexible structures, Aircraft and Spacecraft, Servomechanisms, Load frequency of power systems, guidance, Pulse-width modulation, process control, power converters, digital implementation, and remote vehicle control
- SMC are also being used on neural net learning algorithms, missile autopilot, and of course reconfigurable flight control.

Proof Problem

- Consider the double integrator control law:

$$\ddot{y}(t) = u(t) \quad u(t) = -ky(t)$$

- The pure undamped harmonic motion with ydot(0)=0, y(0)=1, k=4. The phase plane plot of the oscillator is:

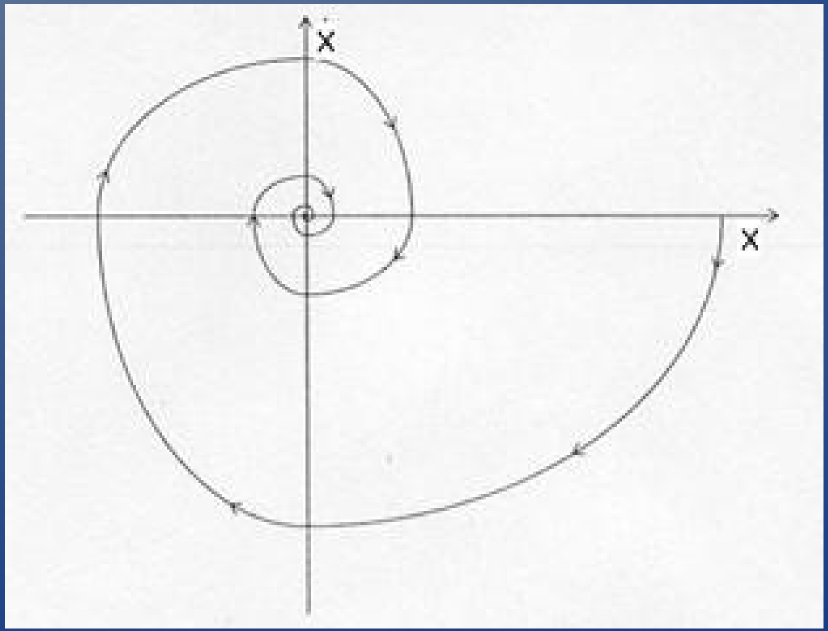


Proof Problem

- Consider instead:

$$u(t) = -k_1 y(t) \ \ if \ \ y\,\dot{y} < 0 \ \ else - k_2 y(t)$$

- For ydot(0)=0, y(0)=0, k1=.5, k2=4 Phase VSC for a lightly damped second order system:

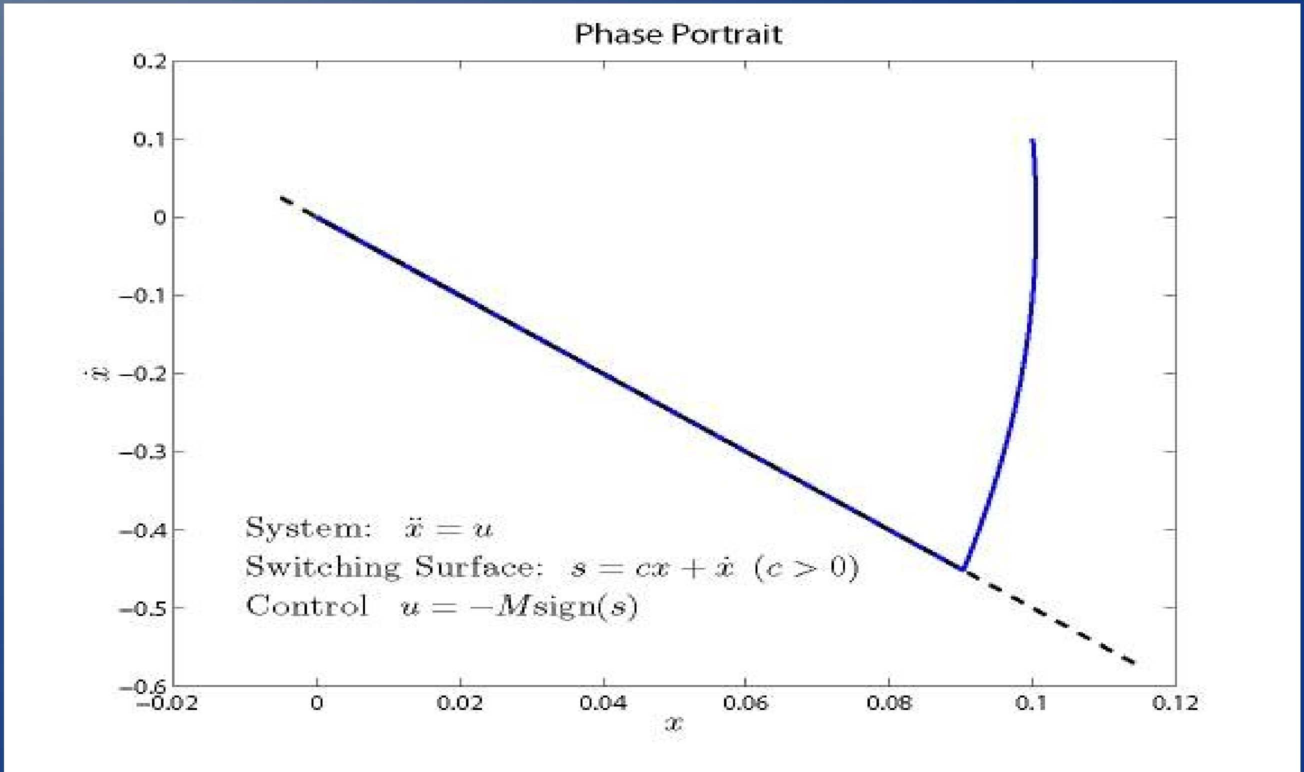


Proof Problem

- Next instead of a quadrant controller consider the switching function and controller where c is a positive scalar: $\sigma(y,\dot{y}) = cy + \dot{y}$

$$u(t) = -1 \quad if\ \sigma(y,\dot{y}) > 0 \quad else\ 1 \quad if\ \sigma(y,\dot{y}) < 0$$

- for c=1 the system behaves like a perfectly damped second order system with phase:



SMC Properties

- Given a state space system with sliding surface (1), with the square matrix SB nonsingular:

$$(1) \quad \bar{\sigma}(\bar{x}) = S\bar{x}$$

- The sliding surface motion given by (2) is of reduced order and the e-values associated with any non-zero e-vector of the system (3) belongs to the null space of the matrix S.

$$(2)\ \dot{\bar{X}}(t) = (I_B - B(Sb)^{-1}S)A\bar{x}(t)\ for\ t \geq t_s \wedge S\bar{x}(t_s) = 0$$

$$(3)\ A_{eq} = (I_n - B(SB)^{-1}S)A$$

- The ideal sliding motion is complete insensitive to the uncertainly functions zeta in (4).

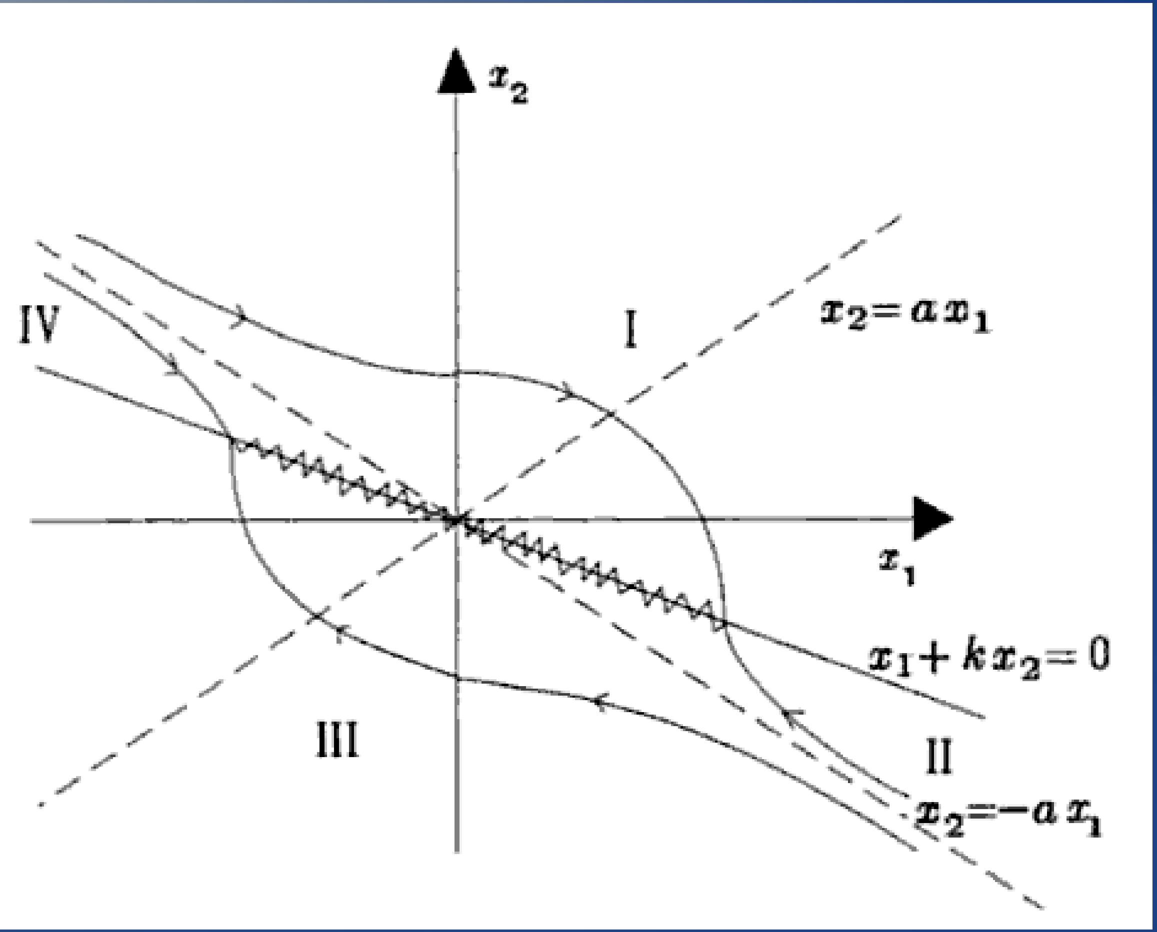
$$(4)\quad \dot{\bar{X}}(t) = A\bar{x}(t) + B\bar{u}(t) + D\bar{\zeta}(t,\bar{x})\ if\ R(D) \in R(B)$$

What does all of this mean?

- The line or hyper-surface that describes sigma=0 defines the transient response of the system
- During sliding, the trajectory dynamics are of lower order than the original model
- While in sliding, the dynamics are solely governed by the parameters that describe sigma=0
- The trajectory of sliding is not inherent in either control structure but a combination thereof.

Summary: The SMC method provides the best tracking results of any of the other methods while automatically guaranteeing the most cost effective progression. This is done *without* the need for parameter identification. SMC is known for being very robust (invariant) to many kinds of uncertainty which is why it is the ideal choice for reconfigurable designs.

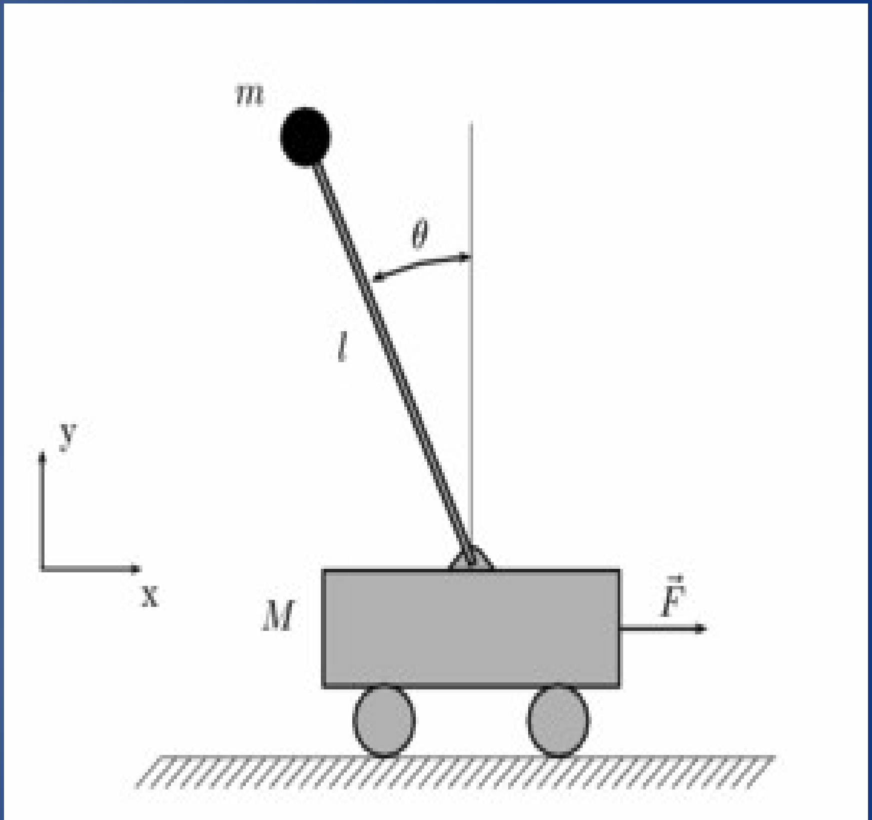
Reachability and Chatter



DESIGN EXAMPLES

Inverted Pendulum on Translating Cart

- System Parameters:
  - Cart Mass : M (3kg)
  - Pendulum Mass : m (.5kg)
  - Pendulum Length : L (.4m)
  - Linear Friction Coeff. Fx: (6kg/s)
  - Angular Friction Coeff. F_th: (.005kgm^2)
- State Variables
  - Cart Position : x
  - Pendulum angle: θ
- Control Inputs
  - Horizontal Force : u
  - Pendulum Torque : τ



$$(M+m)\ddot{x}+F_x\dot{x}+(mL\cos\theta)\ddot{\theta}-mL\dot{\theta}^2\sin\theta=u$$
$$J\ddot{\theta}+F_\theta\dot{\theta}-mLg\sin\theta+(mL\cos\theta)\ddot{x}=\tau$$

Linearized about Theta = 0

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\\ \dot{z}_4\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{(-m^2L^2g)}{[J(M+m)+m^2L^2]} & \frac{(-JF_x)}{[J(M+m)+m^2L^2]} & \frac{(mLF_\theta)}{[J(M+m)+m^2L^2]}\\ 0 & \frac{[(M+m)mLg]}{[J(M+m)+m^2L^2]} & \frac{(mLF_x)}{[J(M+m)+m^2L^2]} & \frac{[-(M+m)F_\theta]}{[J(M+m)+m^2L^2]}\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ \frac{J}{[J(M+m)+m^2L^2]} & \frac{-mL}{[J(M+m)+m^2L^2]}\\ \frac{-mL}{[J(M+m)+m^2L^2]} & \frac{(M+m)}{[J(M+m)+m^2L^2]}\end{bmatrix}\begin{bmatrix}u\\ \tau\end{bmatrix}$$

- Initially consider SISO tau =0:

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\\ \dot{z}_4\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -1.6345 & -2 & 0.0042\\ 0 & 28.6037 & 5 & -0.0729\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0.3333\\ -0.8333\end{bmatrix}\begin{bmatrix}u\end{bmatrix}$$

Design Procedure – Code Only

- Design the Sliding Surface
- Change coordinates of given system to regular form
  - Perform QR decomposition on the input distribution matrix to get T_r
  - Obtain A_reg, B_reg using T_r
  - Obtain matrix sub-blocks in the regular form equations
  - Use linear quadratic cost function to design the switching function matrix coefs.
  - Transform weighting matrix to regular form coordinates
  - Compute $u_{eq} = -(SB)^{-1} SA$
  - Finally design parameter gain p

For the sake of comparison, this plot shows the results of a simulation using a standard LQR regulator using the same weights given above. As expected, an LQR controller does a good job. This same controller is used on the full non-linear system simulation and achieves exactly the same results. That plot is not given because it looks essentially the same as this one.

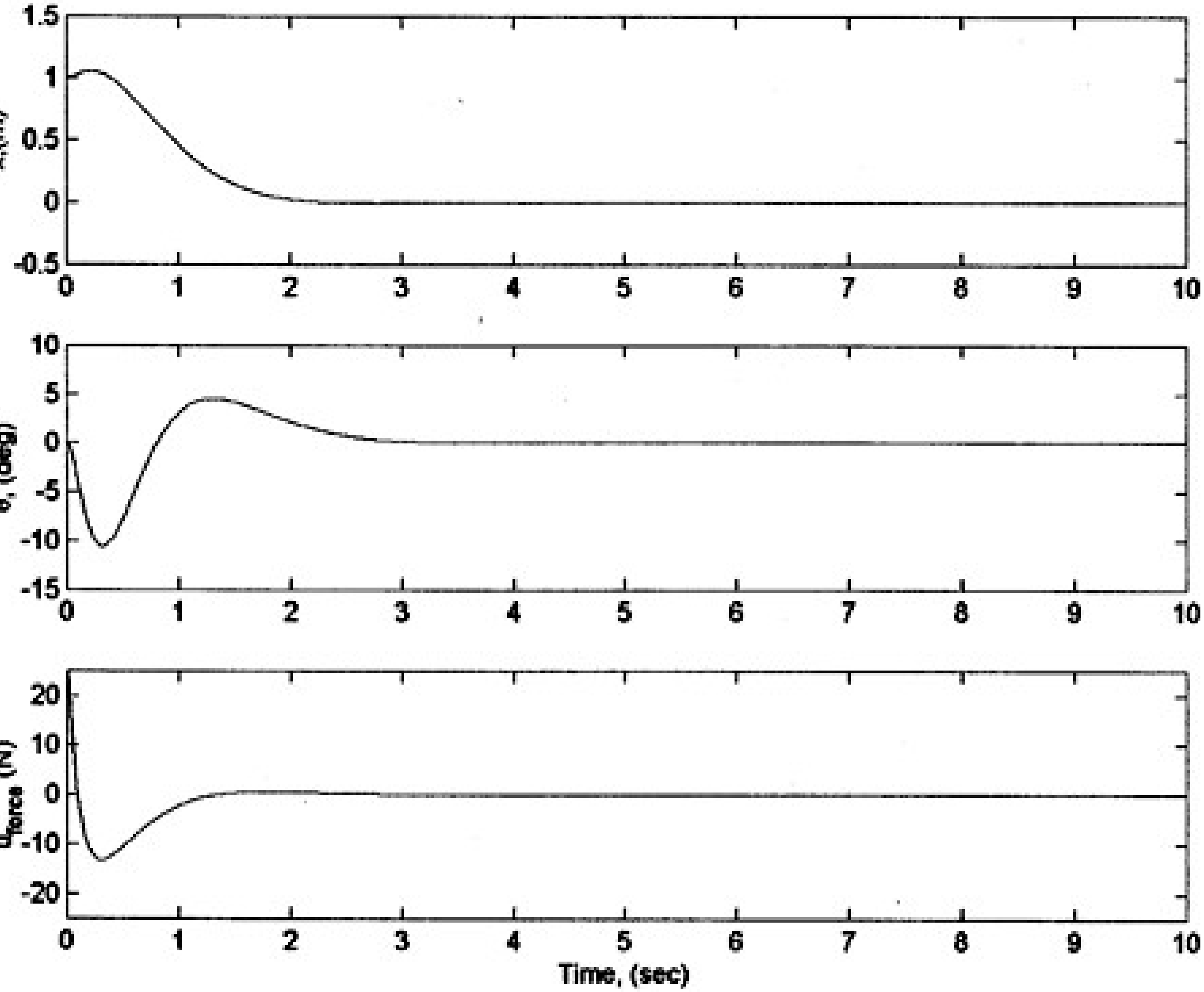


Figure B-13: LQR Regulator (Linear Model)

This SMC controller uses only the equivalent control—it has no discontinuous control element. Note, U_{eq} will not reach the sliding mode by itself. A typical discontinuous element is used during the reaching phase ($0 < t < 0.15$ sec) and turned off as soon as the sliding mode is reached. As expected, U_{eq} maintains the sliding mode for this case because the simulation model is exactly the same as the design model, and there is no noise. When this same control is applied to the non-linear simulation (or if noise is present), stable control is lost.

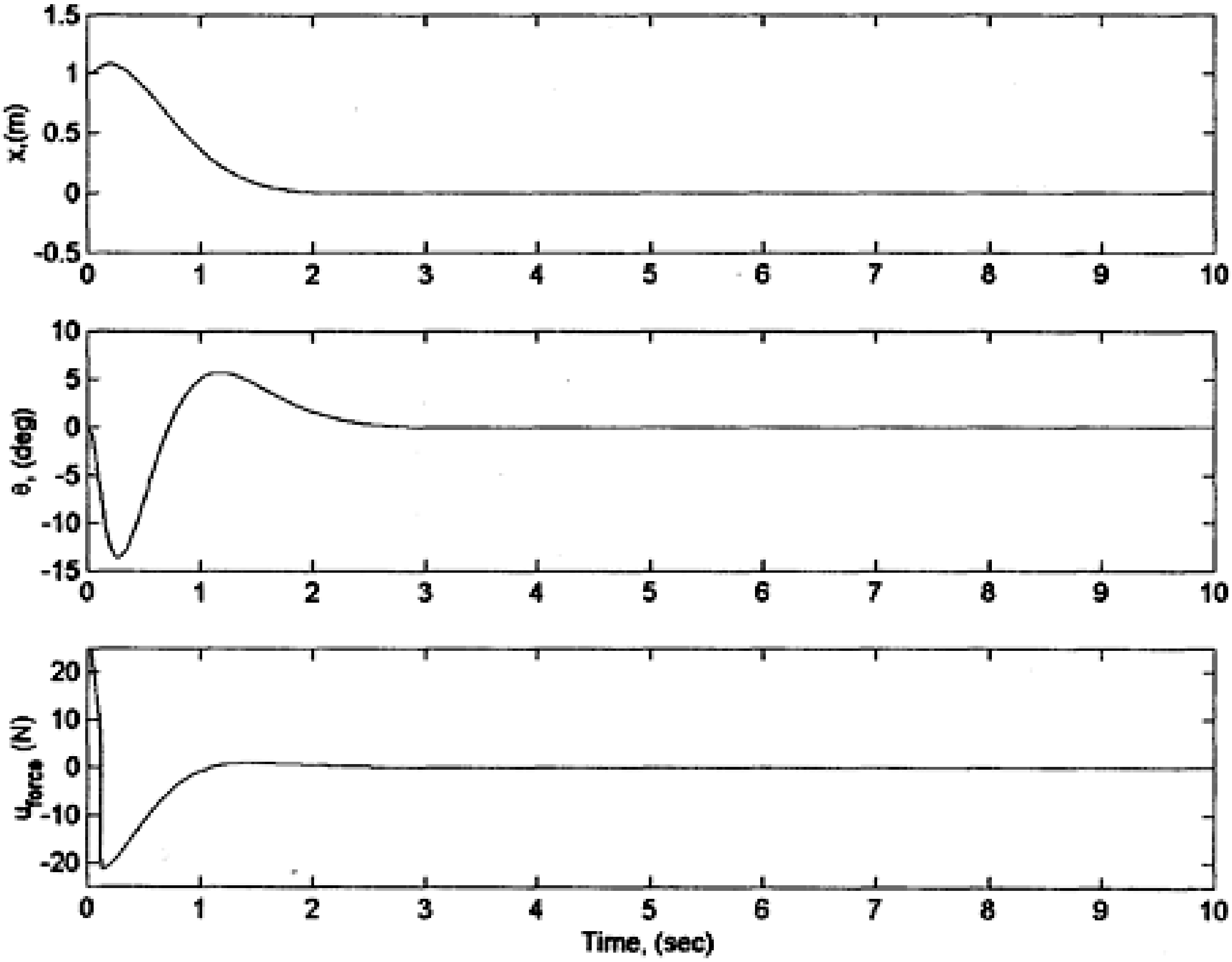


Figure B-14: SMC Regulator (Linear Model) with $u = U_{eq}$

This SMC controller uses no equivalent control. Notice the high activity of the control (near infinite frequency). This is because this case uses a fairly small boundary layer. As the boundary layer approaches zero (ideal sliding mode), the frequency of the control activity becomes infinite. This same controller is used on the full non-linear system simulation and achieves exactly the same results. That plot is not given because it looks essentially the same as this one.

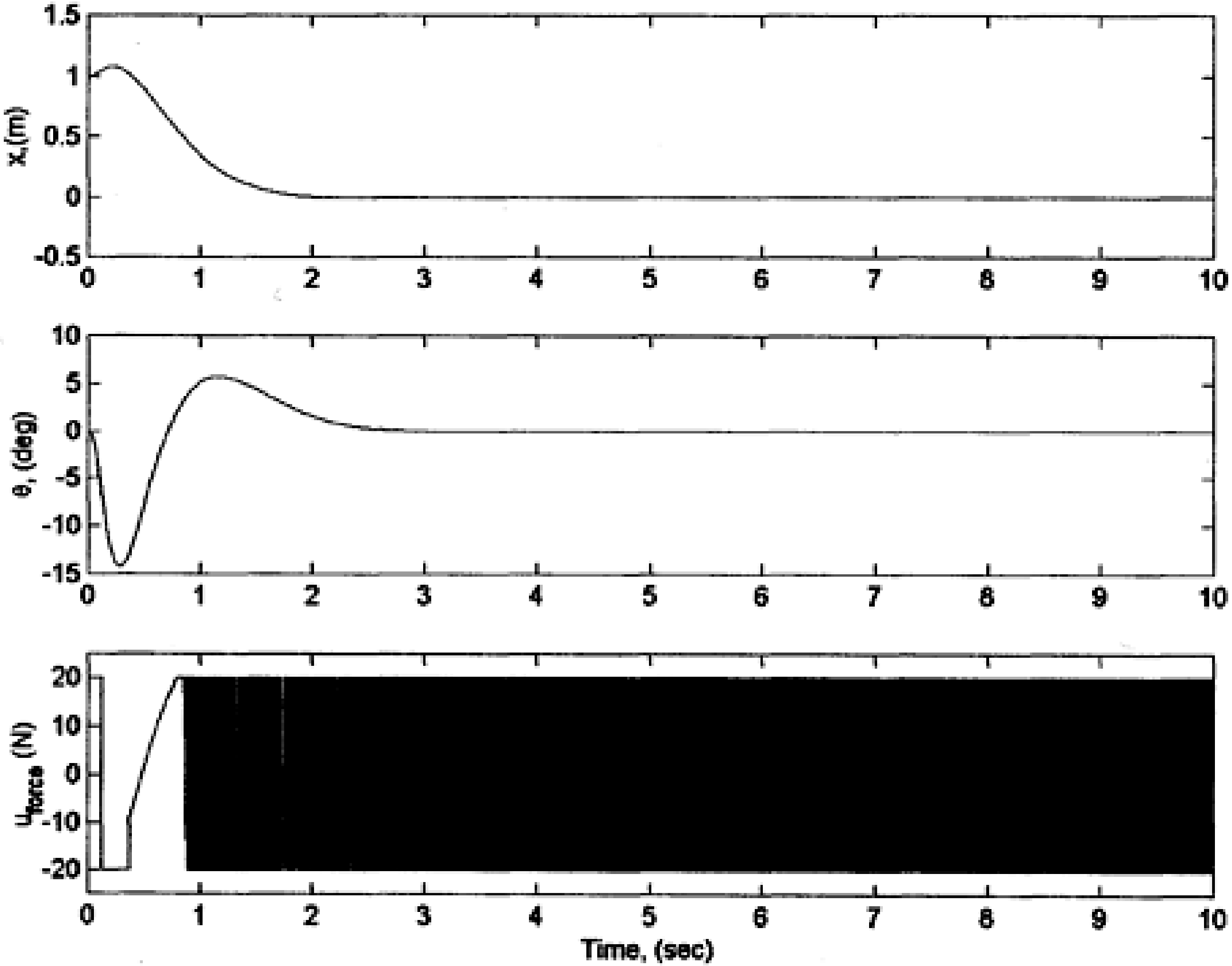


Figure B-15: SMC Regulator (Linear Model) with $u = 20\ \mathrm{sat}(\sigma/0.005)$

This SMC controller uses both the equivalent control and the discontinuous element. Note that the magnitude of the discontinuous part is considerably less than that required for the previous example. That is because the continuous equivalent control is doing a lot of the work. The boundary layer is very small in this case to show the effect of approaching the ideal sliding mode. This same controller is used on the full non-linear system simulation and achieves exactly the same results. That plot is not given because it looks essentially the same as this one.

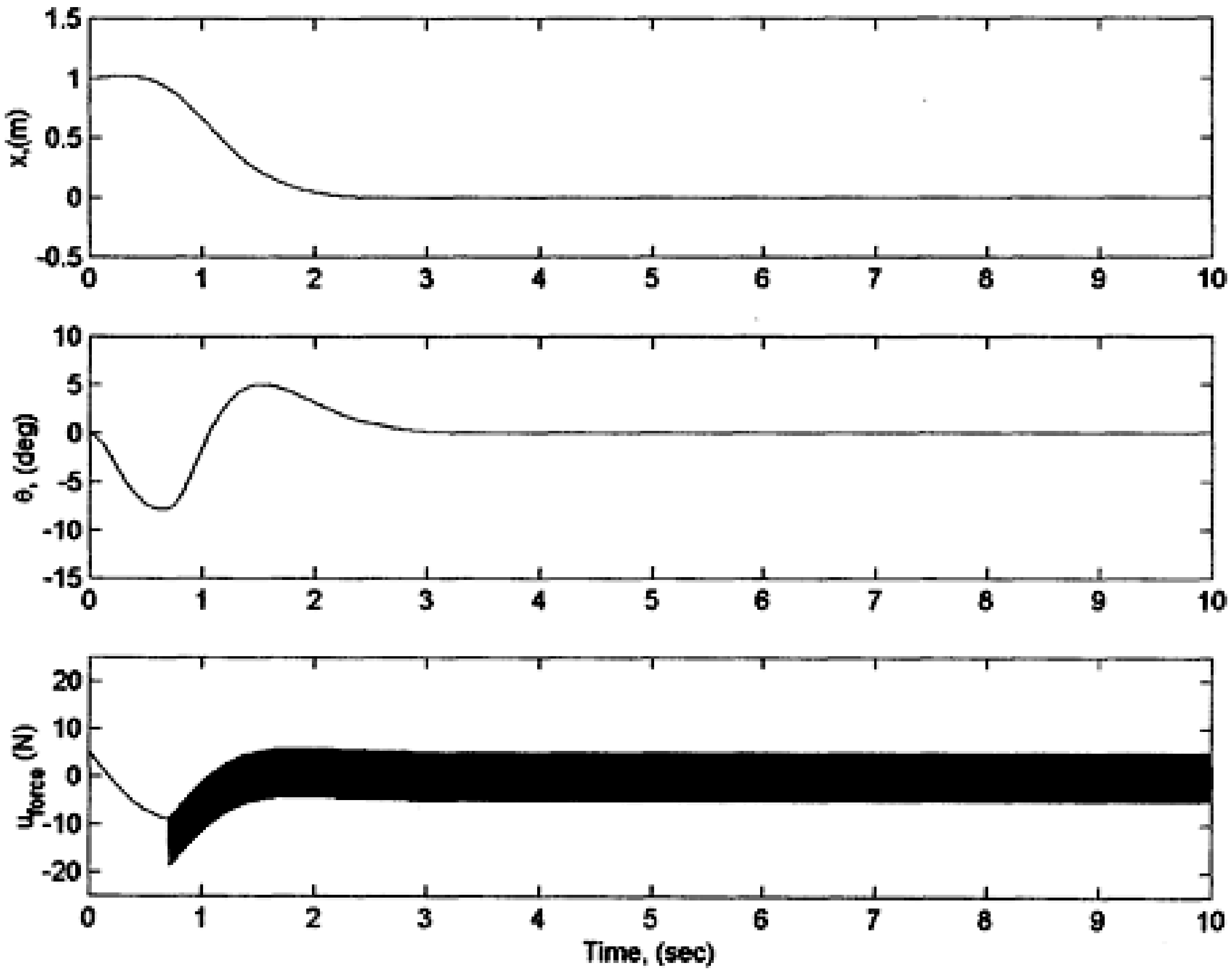


Figure B-16: SMC Regulator (Linear Model) with $u = U_{eq} + 5\ \mathrm{sat}(\sigma/0.0001)$

This SMC controller uses both the equivalent control and the discontinuous element. This case differs from the previous case in the size of the boundary layer. In this case the boundary layer is increased to the point that control appears completely continuous. If the boundary layer is increased too much, the performance becomes noticeably degraded. This same controller is used on the full non-linear system simulation and achieves exactly the same results. That plot is not given because it looks essentially the same as this one.

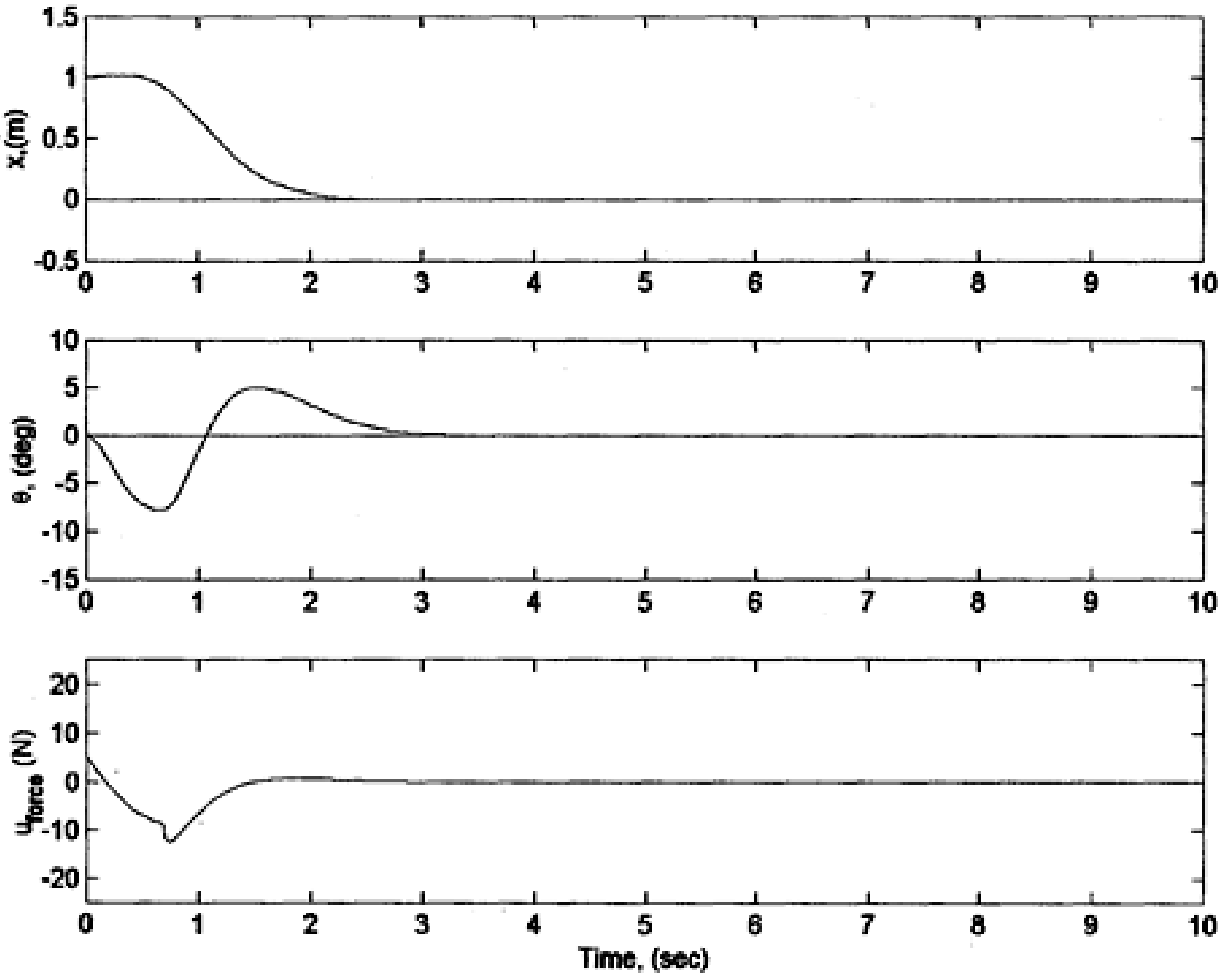


Figure B-17: SMC Regulator (Linear Model) with $u = U_{eq} + 5\ \mathrm{sat}(\sigma/0.1)$

This is the LQR controller using the same weights as previously. White Gaussian noise is injected at the control input. A disturbance bias of 10 is added to the control input at $t = 5$ sec. Pendulum angle tracking is noisy. Cart position has a noticeable bias after 5 sec.

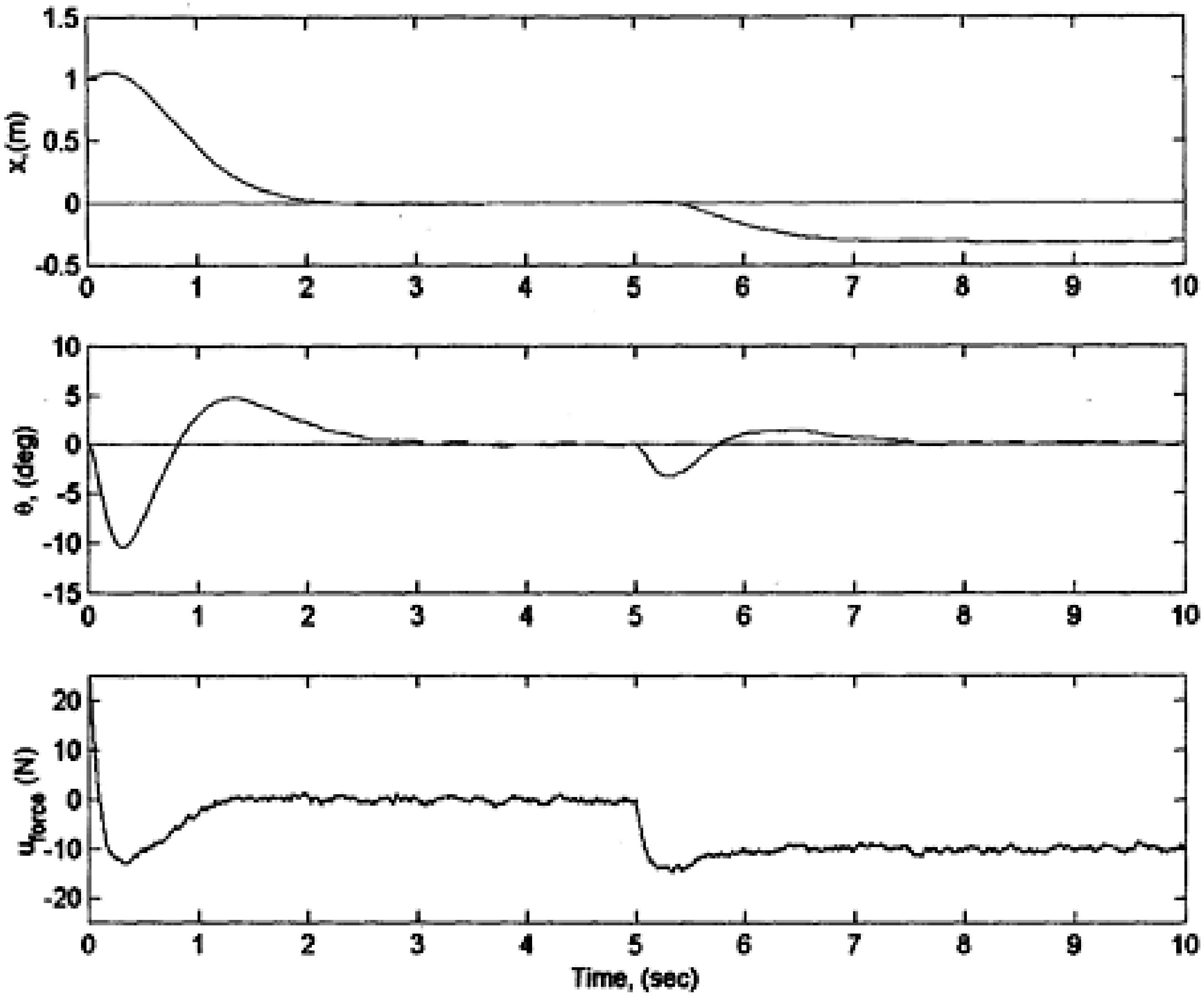


Figure B-18: LQR Regulator (Non-linear System) with Control Input Disturbances

This is the SMC controller using the control law: $u = U_{eq} + 50\ \mathrm{sat}(\sigma/0.01)$. White Gaussian noise is injected at the control input. A disturbance bias of 10 is added to the control input at $t = 5$ sec. Pendulum angle and cart position regulation is very good. A fairly small boundary layer is used--the control effort can be smoothed out some by using a larger boundary layer. This demonstrates SMC's ability to completely reject matched disturbances.

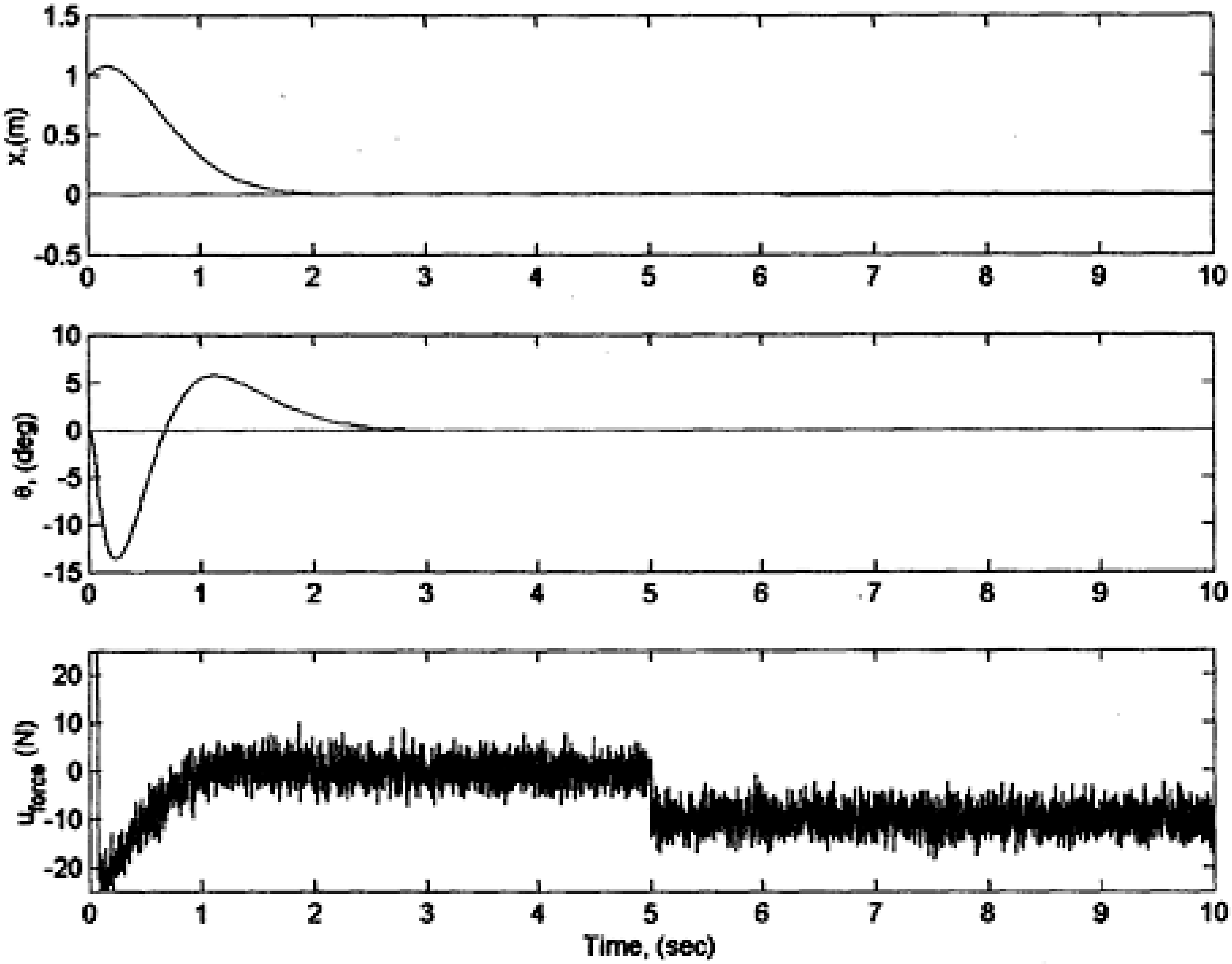


Figure B-19: SMC Regulator (Non-linear System) with Control Input Disturbances

This plot shows the SMC controller with: $u = 50\,\mathrm{sat}\left(\frac{\sigma_x}{0.4}\right)$ and $\tau = 10\,\mathrm{sat}\left(\frac{\sigma_\theta}{0.4}\right)$. A non-linear system simulation is run with no system failure. Very nice decoupled tracking is demonstrated. The baseline LQR simulation looks the same.

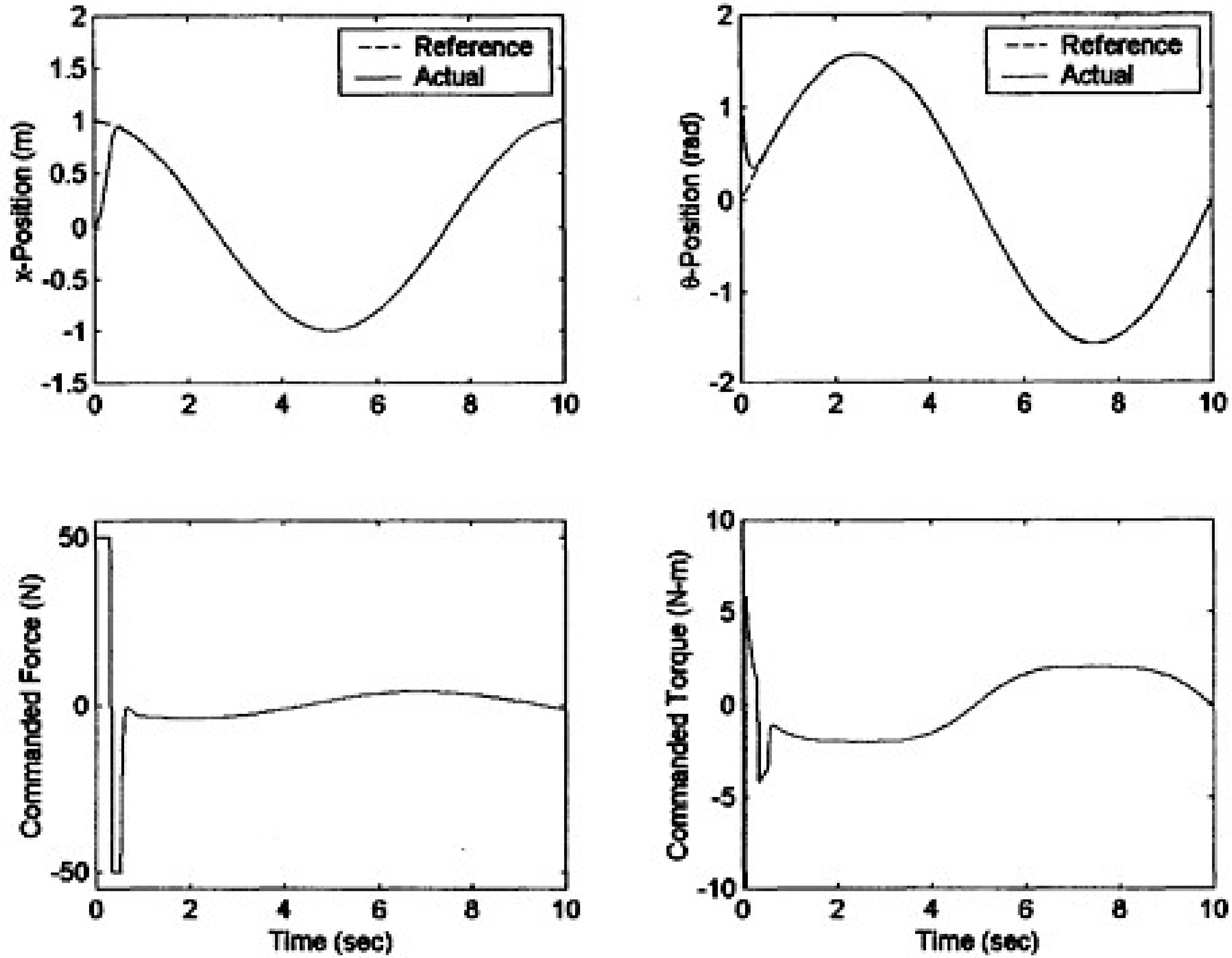


Figure B-20: Decoupled SMC Tracking, Baseline Case

This plot shows the LQR controller. The disturbance bias at 5 sec has magnitude of 50. A non-linear system simulation is run with system failure at 3 sec. Tracking performance is severely degraded.

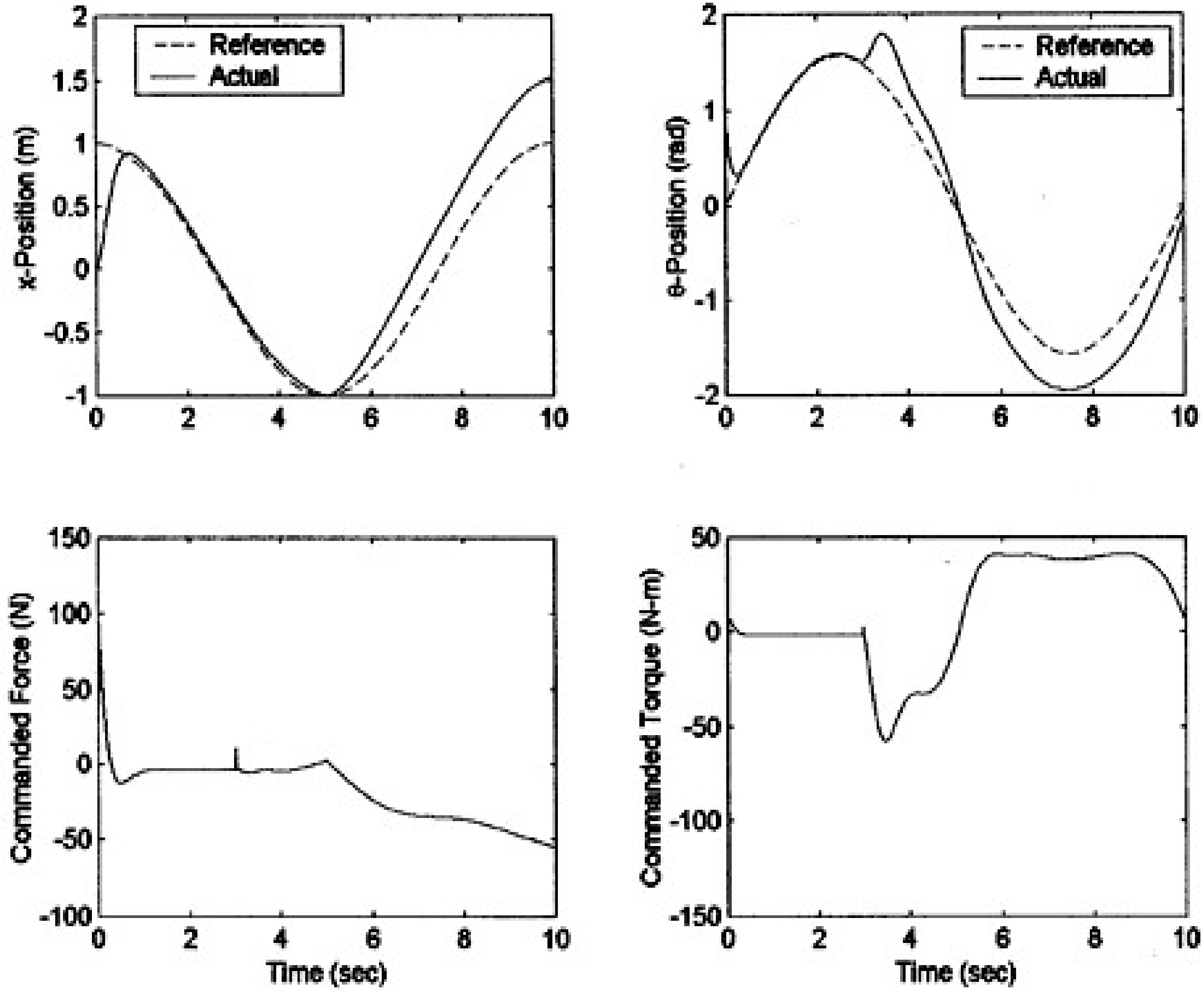


Figure B-21: LQR Tracking with System Failure

This plot shows the SMC controller with: $u = 50\,\mathrm{sat}\left(\dfrac{\sigma_x}{0.4}\right)$ and $\tau = 10\,\mathrm{sat}\left(\dfrac{\sigma_\theta}{0.4}\right)$. The disturbance bias at 5 sec has magnitude of 50. A non-linear system simulation is run with system failure at 3 sec. Note the controller is still tracking x after the failure at 3 sec (although some degradation is apparent), but the disturbance bias at 5 sec causes x to diverge. The significance of this plot is that it demonstrates that if the disturbance level is higher than designed for, the SMC will not be able to stabilize the system—and, in fact, may drive a statically stable state unstable.

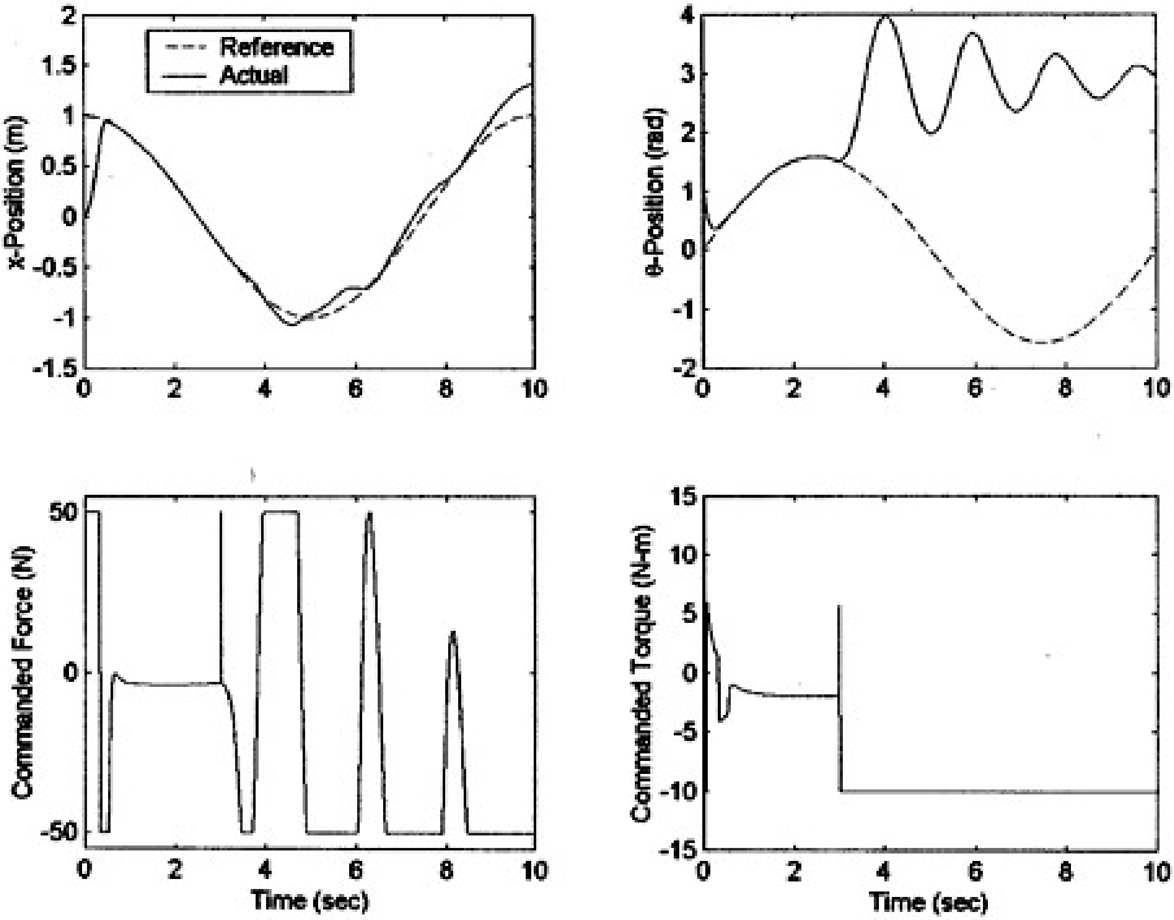


Figure B-23: SMC Tracking with System Failure (Insufficient ρ_x & ρ_θ)

This plot shows the SMC controller with: $u = 175\,\text{sat}\left(\frac{\sigma_x}{0.4}\right)$ and $\tau = 10\,\text{sat}\left(\frac{\sigma_\theta}{0.4}\right)$. The disturbance bias at 5 sec has magnitude of 50. A non-linear system simulation is run with system failure at 3 sec. Note the controller is tracking x throughout, but θ breaks tracking shortly after the system failure. The significance of this plot is that is demonstrates the ability of the controller to track one state even though another (coupled) state is not on its sliding mode.

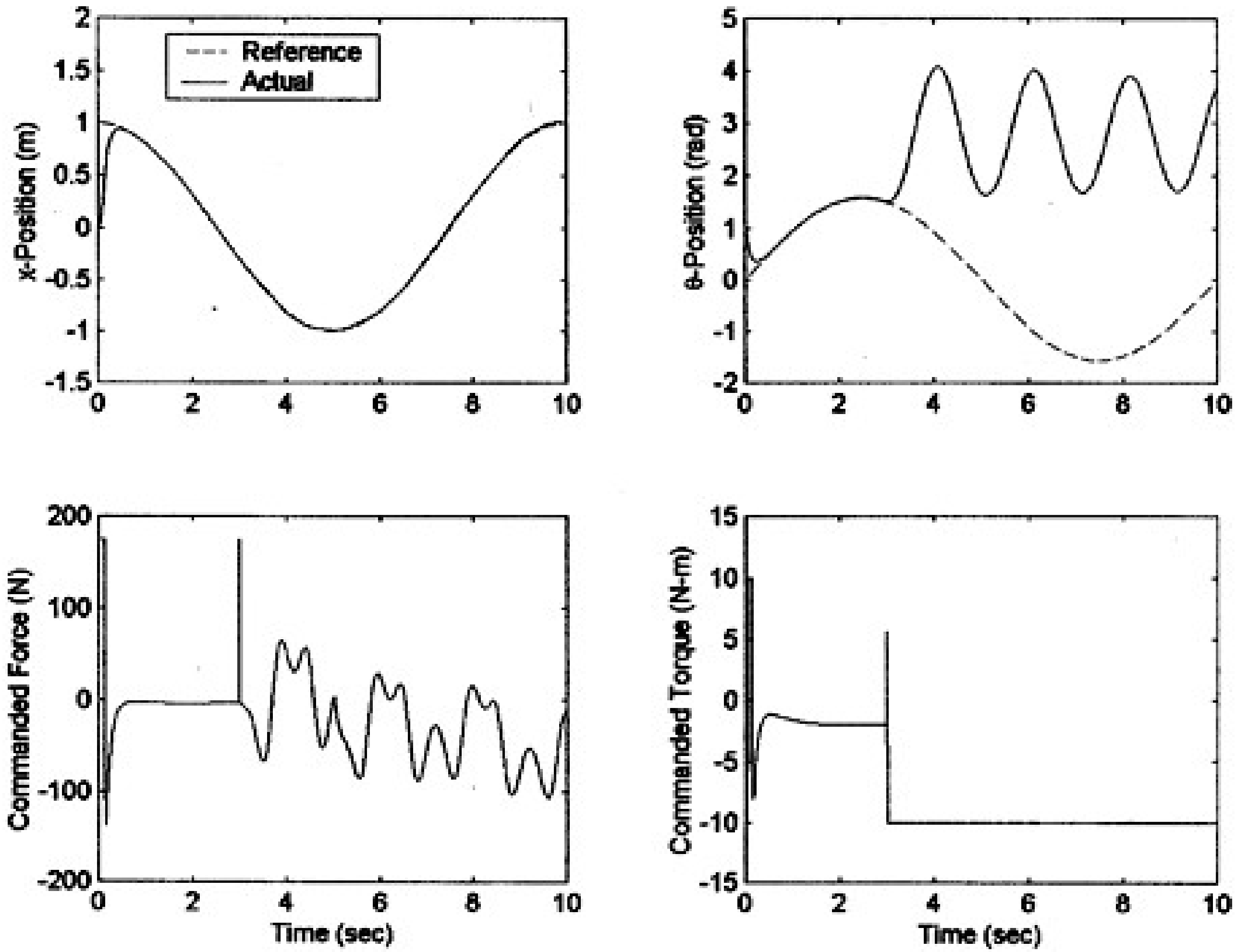


Figure B-24: SMC Tracking with System Failure (Insufficient ρ_θ)

This plot shows the SMC controller with: $u = 175\,\mathrm{sat}\left(\frac{\sigma_x}{0.4}\right)$ and $\tau = 75\,\mathrm{sat}\left(\frac{\sigma_\theta}{0.4}\right)$. The disturbance bias at 5 sec has magnitude of 50. A non-linear system simulation is run with system failure at 3 sec. Very nice decoupled tracking is demonstrated. Notice the gains on the discontinuous control elements are higher than the baseline case. This is because a larger value is needed to provide the reaching condition in the face of the increased uncertainty.

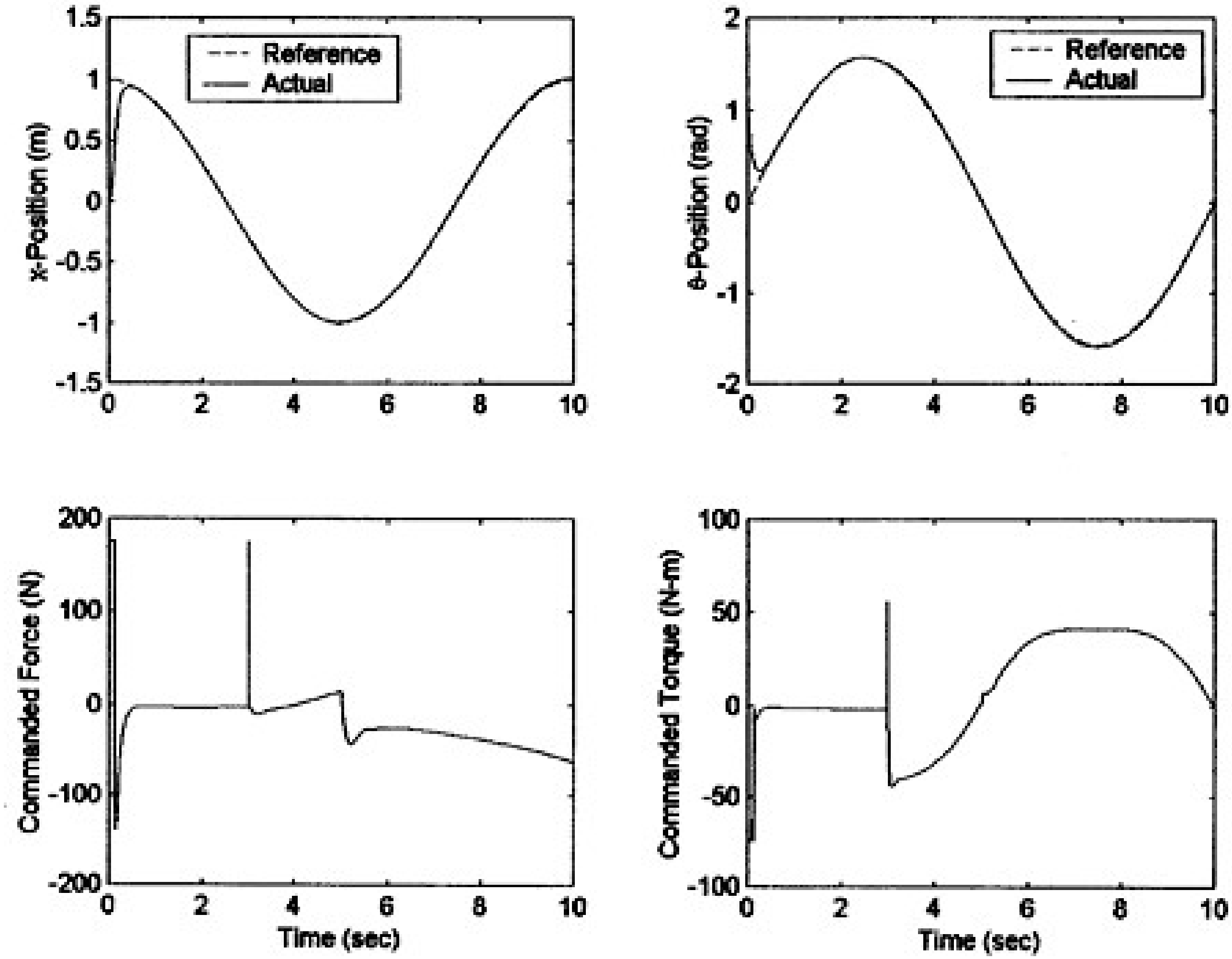


Figure B-22: SMC Tracking with System Failure

S-61 Helicopter Matlab-Simulink

- Psueodo-Sliding Mode Design Approach
  - 1) Vehicle Model is obtained. Actuator dynamics and flexible modes such as rotor degrees of freedom are ignored at this stage.
  - 2) A square control structure is identified with specific desired command/response relations
  - 3) Sliding manifolds for each control element are selected and interpreted in the frequency domain as compensation elements
  - 4) The existence of sliding behavior is verified
  - 5) A boundary layer is introduced into each control channel to eliminate the infinite-frequency control switching
  - 6) Hitherto neglected actuator and flexible modes are reintroduced as parasitic dynamics. Typically this results in instability of the system.
  - 7) Asymptotic observers are introduced to accommodate parasitic dynamics
  - 8) Computer Simulations

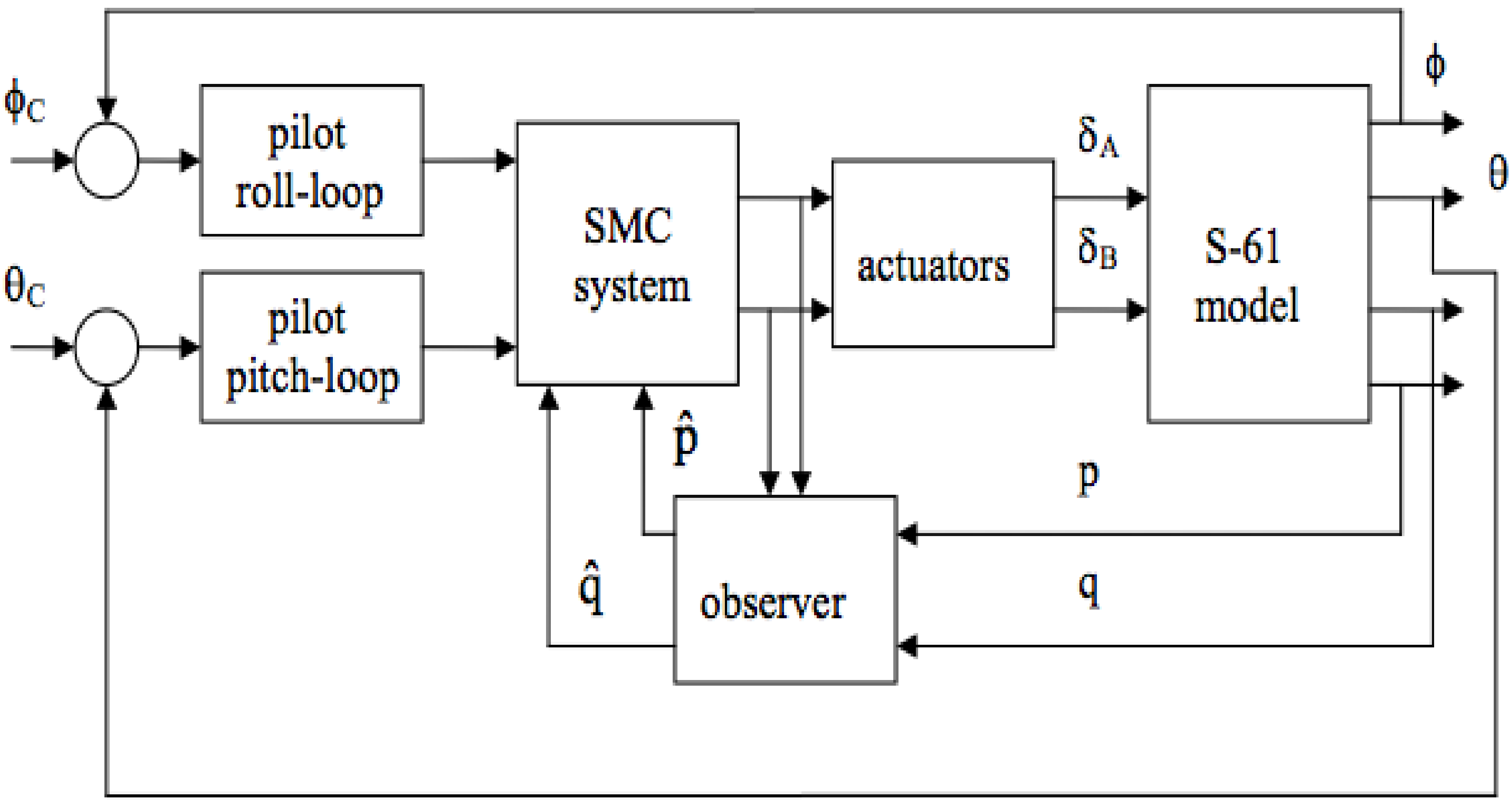


Figure 1 The pilot/vehicle system

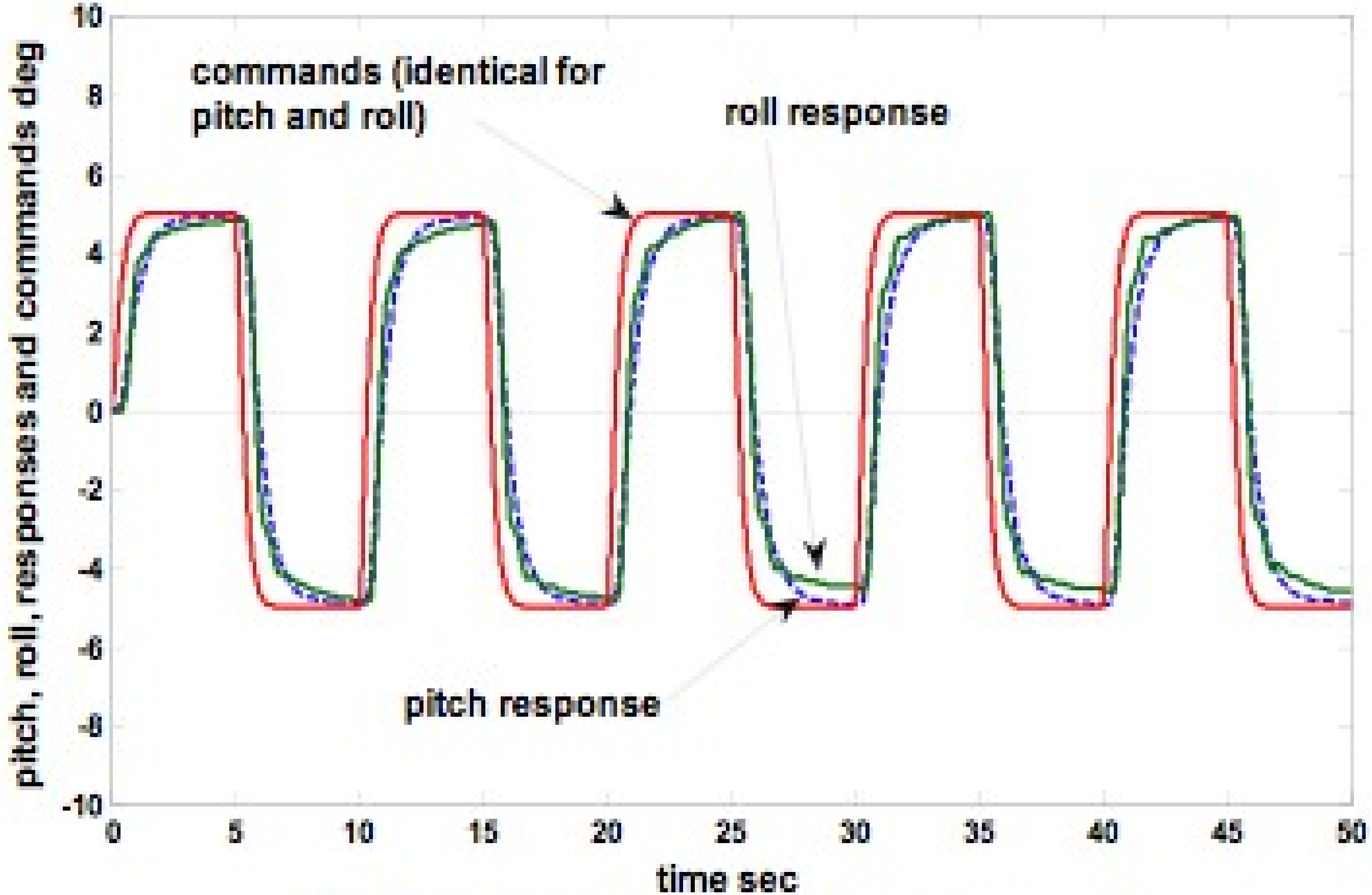


Figure 2 Pilot/vehicle performance

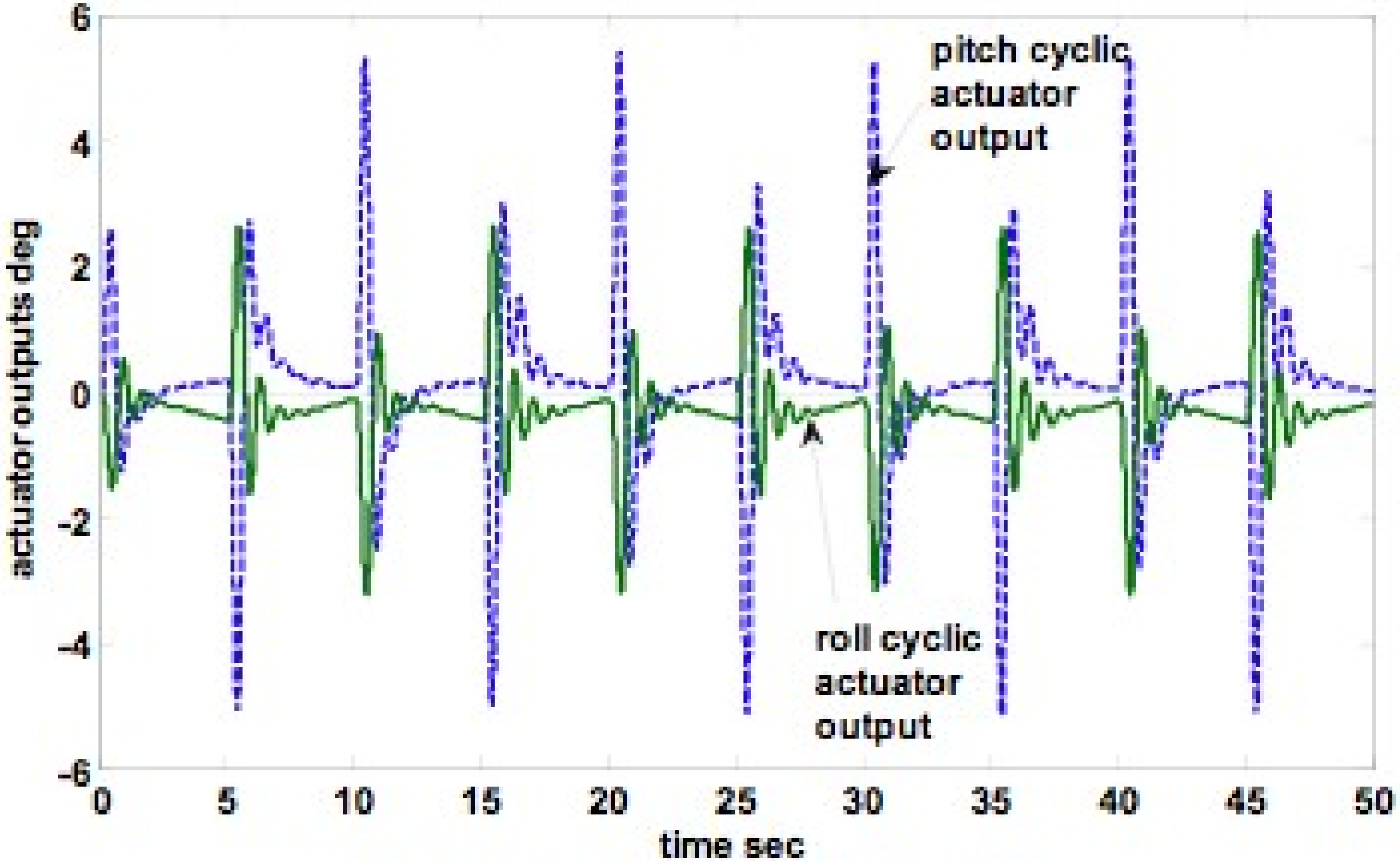


Figure 3 Actuator outputs

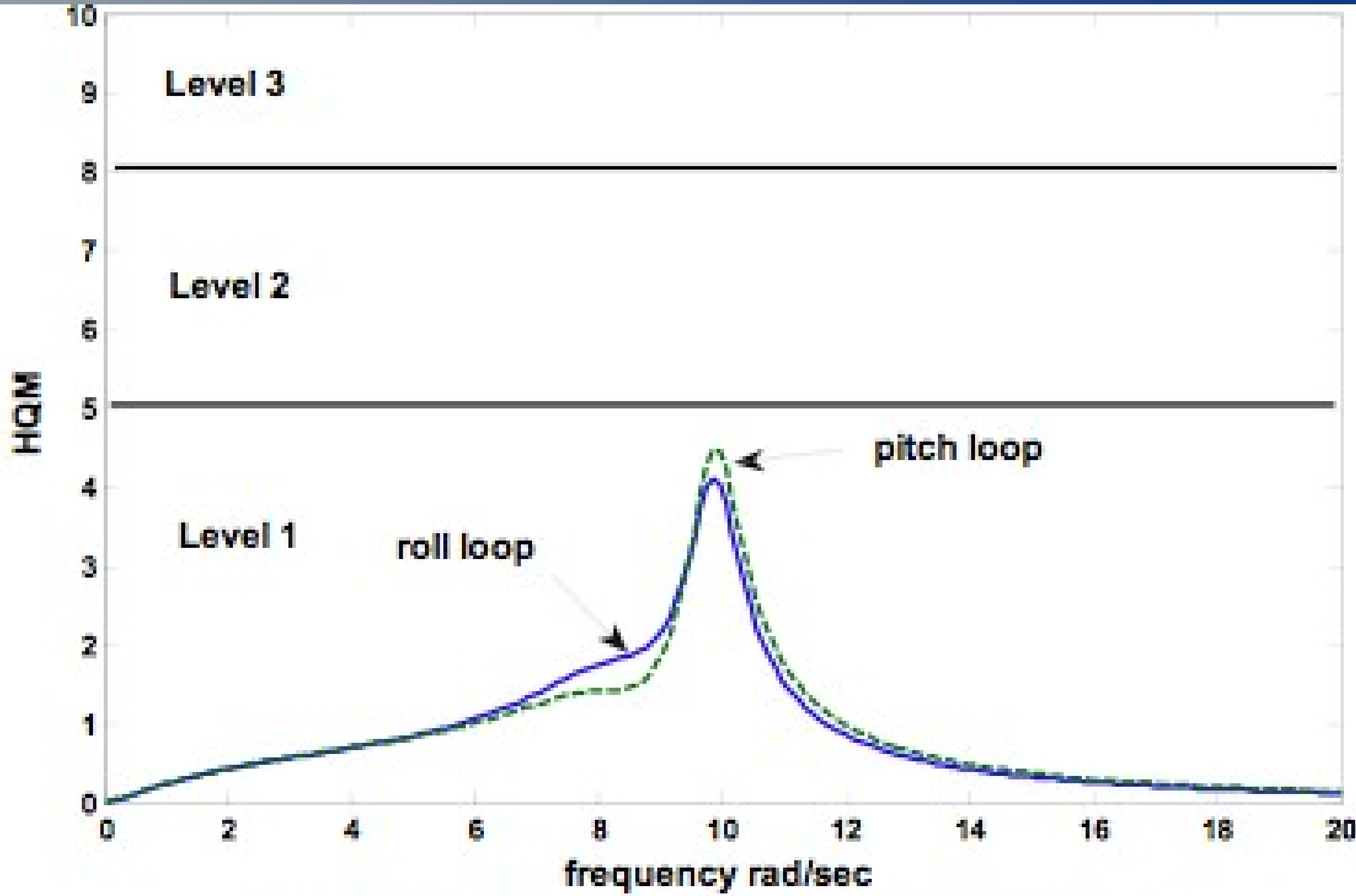


Figure 4 Handling qualities prediction

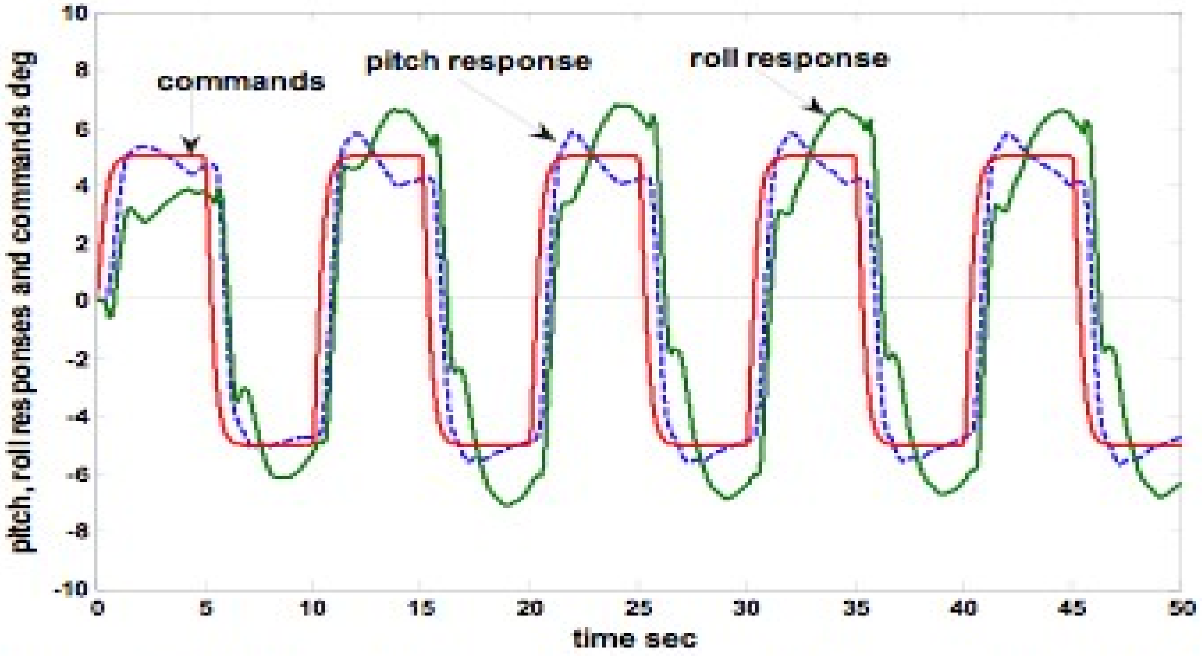


Figure 5 Pilot/vehicle performance, "damaged" vehicle

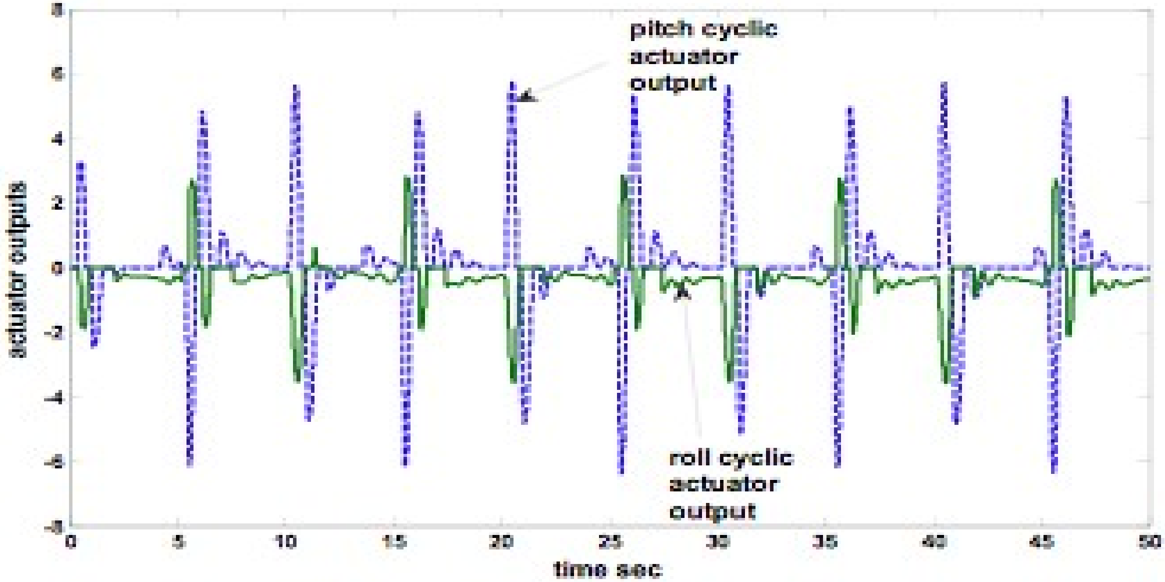


Figure 6 Actuator outputs, "damaged" vehicle

References

- Scott G. Wells PhD Dissertation
- SMC in Engineering Text
- Robust Control by R. Hess and A. Pechner