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Gyrostats – an introduction

Dale Lukas Peterson

University of California Davis

April 21, 2011

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   - Examples
2. Theory
   - Nomenclature
   - Useful formulas
   - An example
3. Conclusions

Definition of a Gyrostat

A system of rigid bodies and/or particles whose

- System center of mass location is independent of configuration
- System central inertia tensor is independent of configuration

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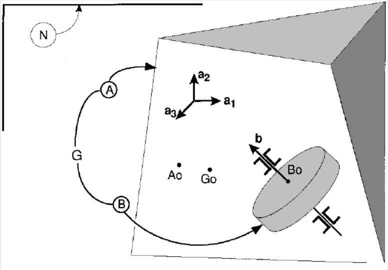
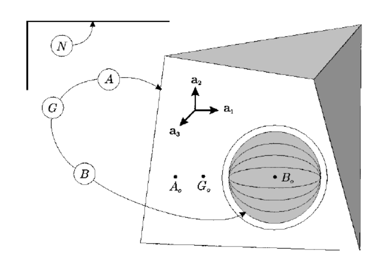
Types of gyrostats

Cylindrical gyrostat



Spherical gyrostat



Everyday examples

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- Toy Gyroscopes
- Unicycles (symmetric wheel)
- Bicycles (two unicycles connected by a hinge)
- Helicoptors (main rotor and tail rotor)
- Flywheels
- Spherical dampers (satellite)

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Nomenclature

- "Carrier" – A
  - mass m_A
  - Central inertia tensor $\boldsymbol{I}^{A/A_o}$
  - Angular velocity ${}^N\boldsymbol{\omega}^A$
- "Rotor" – B
  - mass m_B
  - Symmetry axis $\boldsymbol{b}$
  - Central inertia tensor
    - $\boldsymbol{I}^{B/B_o} = K\boldsymbol{U} + (J - K)\boldsymbol{bb}$ (cylindrical rotor)
    - $\boldsymbol{I}^{B/B_o} = I\boldsymbol{U}$ (spherical rotor)
  - ${}^A\boldsymbol{\omega}^B = \Omega\boldsymbol{b}$
- "Gyrostat" – G, mass m_G, Central inertia tensor $\boldsymbol{I}^{G/G_o}$
- "Rigid Gyrostat" – RG, mass m_G, Central inertia tensor $\boldsymbol{I}^{G/G_o}$, *same angular velocity as* A

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Mass and Inertia Relationships

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- $m_G = m_A + m_B$
- $\boldsymbol{I}^{G/G_o} = \boldsymbol{I}^{A/A_o} + \boldsymbol{I}^{A_o/G_o} + \boldsymbol{I}^{B/B_o} + \boldsymbol{I}^{B_o/G_o}$
- $\boldsymbol{I}^{CG/G_o} \triangleq \boldsymbol{I}^{G/G_o} - J\boldsymbol{bb}$
- $\boldsymbol{I}^{SG/G_o} \triangleq \boldsymbol{I}^{G/G_o} - I\boldsymbol{bb}$

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Angular Momentum

Free rotors (traditional) – $\omega_s \triangleq {}^A\boldsymbol{\omega}^B \cdot \boldsymbol{b}$

$$
\begin{aligned}
{}^N\boldsymbol{H}^{RG/G_o} &\triangleq \boldsymbol{I}^{G/G_o} \cdot {}^N\boldsymbol{\omega}^A \\
{}^N\boldsymbol{H}^{G/G_o} &= {}^N\boldsymbol{H}^{RG/G_o} + J\omega_s\boldsymbol{b}
\end{aligned}
$$

Free rotors ("efficient") – $\omega_s \triangleq {}^N\boldsymbol{\omega}^B \cdot \boldsymbol{b}$

$$
\begin{aligned}
{}^N\boldsymbol{H}^{CG/G_o} &\triangleq \boldsymbol{I}^{CG/G_o} \cdot {}^N\boldsymbol{\omega}^A \\
{}^N\boldsymbol{H}^{G/G_o} &= {}^N\boldsymbol{H}^{CG/G_o} + J\omega_s\boldsymbol{b}
\end{aligned}
$$

Inertia Torque

Free rotors (traditional) – $\omega_s \triangleq {}^A\boldsymbol{\omega}^B \cdot \boldsymbol{b}$

$$
\begin{aligned}
{}^N\boldsymbol{T}^{RG} &\triangleq \boldsymbol{I}^{G/G_o} \cdot {}^N\boldsymbol{\alpha}^A + {}^N\boldsymbol{\omega}^A \times \boldsymbol{I}^{G/G_o} \cdot {}^N\boldsymbol{\omega}^A \\
{}^N\boldsymbol{T}^{G} &= {}^N\boldsymbol{T}^{RG} + J(\dot{\omega}_s\boldsymbol{b} + {}^N\boldsymbol{\omega}^A \times \omega_s\boldsymbol{b})
\end{aligned}
$$

Free rotors ("efficient") – $\omega_s \triangleq {}^N\boldsymbol{\omega}^B \cdot \boldsymbol{b}$

$$
\begin{aligned}
{}^N\boldsymbol{T}^{CG} &\triangleq \boldsymbol{I}^{CG/G_o} \cdot {}^N\boldsymbol{\alpha}^A + {}^N\boldsymbol{\omega}^A \times \boldsymbol{I}^{CG/G_o} \cdot {}^N\boldsymbol{\omega}^A \\
{}^N\boldsymbol{T}^{G} &= {}^N\boldsymbol{T}^{CG} + J(\dot{\omega}_s\boldsymbol{b} + {}^N\boldsymbol{\omega}^A \times \omega_s\boldsymbol{b})
\end{aligned}
$$

Inertia Torque of G in N

$$
\boldsymbol{T}^* \triangleq -{}^N\boldsymbol{T}^G
$$

Example

Traditional

Carrier A

- mass m_A
- $\boldsymbol{I}^{A/A_o} = I_{xx}\boldsymbol{a}_x\boldsymbol{a}_x + I_{yy}\boldsymbol{a}_y\boldsymbol{a}_y + I_{zz}\boldsymbol{a}_z\boldsymbol{a}_z + I_{xz}\boldsymbol{a}_x\boldsymbol{a}_z + I_{xz}\boldsymbol{a}_z\boldsymbol{a}_x$

Rotor B

- mass m_B
- $\boldsymbol{I}^{B/B_o} = I\boldsymbol{a}_x\boldsymbol{a}_x + J\boldsymbol{a}_y\boldsymbol{a}_y + I\boldsymbol{a}_z\boldsymbol{a}_z$

Geometry $\boldsymbol{r}^{A_o/B_o} = x\boldsymbol{a}_x + z\boldsymbol{a}_z$

New

Gyrostat G

- mass m_G
- $\boldsymbol{I}^{G/G_o} = I_{Gxx}\boldsymbol{a}_x\boldsymbol{a}_x + I_{Gyy}\boldsymbol{a}_y\boldsymbol{a}_y + I_{Gzz}\boldsymbol{a}_z\boldsymbol{a}_z + I_{Gxz}\boldsymbol{a}_x\boldsymbol{a}_z + I_{Gxz}\boldsymbol{a}_z\boldsymbol{a}_x$
- Rotor spin moment of inertia J

Geometry $\boldsymbol{r}^{B_o/G_o} = x_g\boldsymbol{a}_x + z_g\boldsymbol{a}_z$

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To minimize complexity of motion equations:

- For derivation of motion equations, define mass and inertia scalars in a gyrostat framework, rather than defining these scalars for each individual body
- Introduce geometric scalars which locate mass centers relative to the gyrostat mass center
- For free moving rotors, use generalized speeds which measure the component(s) of angular velocity about the axis of rotation, *relative to the inertial frame*
- For rotors with a specified spin rate, use generalized speeds which measure the component(s) of angular velocity about the axis of rotation, *relative to the carrier frame*

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