Anemometer Calibration Uncertainty

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Accurate wind measurements are critical in evaluating wind turbine power performance and site assessment. In a turbine power performance evaluation, wind speed readings are matched with corresponding turbine power measurements to produce a power curve for the turbine. For site assessment, the distribution of measured wind speed is used to determine the predicted annual energy production from the wind. Since wind power is proportional to the cube of the wind speed, a small error in the wind measurement could translate to a much greater error in the predicted wind power, which emphasizes the importance of having accurate wind speed readings. To acquire such precision in wind data, it is recommended that individually calibrated anemometers be employed. With these calibrations, it is also recommended that the uncertainty in the calibration be reported so that it may be used not only in the overall uncertainty for turbine power curves and site assessments, but also in improving the performance of an anemometer. A method of presenting calibration uncertainty is defined in the standard IEC 61400-12-1. However, the standard only refers to the measurement uncertainty of the reference wind speed from the particular test facility. It does not include the uncertainty in the anemometer linear transfer function and the errors directly made by the anemometer signal. This paper will not only discuss the details of uncertainty reporting as defined by IEC 61400-12-1, but also a method of extending the uncertainty to include the errors when using the linear transfer function and a qualitative description of how to determine the uncertainty in a wind speed measurement in the field.

A. Introduction

Energy production estimates for wind turbine sites are determined based on two sources: 1) wind resource assessment and 2) wind turbine power production(6). In both of these sources, wind speed is one of the critical measures that require a detailed uncertainty analysis. Part of the protocols in site assessment is to conduct field measurements of local wind speeds, which are then used to estimate the potential wind power available at a particular site. Turbine power production is determined based on its power curve where the measured turbine power is a function of measured wind speed (see Figure 1).

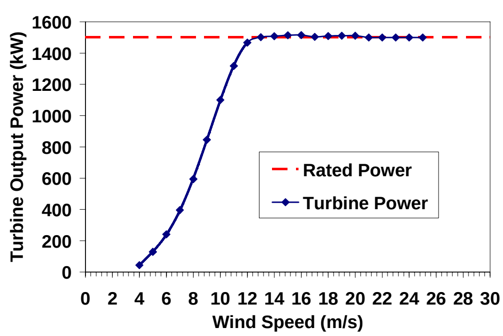

Figure 1: Sample wind turbine power curve.

A common question from end users is whether to use uncalibrated or calibrated anemometers to conduct wind measurements. With uncalibrated anemometers, users resort to applying the manufacturer's published transfer function to convert the anemometer output to wind speed. The published transfer function does have a degree of uncertainty which essentially adds only to the bias of the same model anemometer. From the manufacturer's specification, a level of accuracy is

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also assigned to the anemometer; however, this value also only applies as a bias error. A complete uncertainty analysis investigates not only bias errors but also precision errors, which is related to the repeatability of the wind measurement. It is possible that significant precision errors would be revealed during calibration. In order to gain a better understanding of the uncertainty in the wind speed, calibrated anemometers should be employed. With a calibrated anemometer, error sources of its measurement performance would be accounted for.

In IEC 61400-12-1, a sample uncertainty calculation is provided for a wind tunnel facility that uses a Pitot tube system to measure the reference wind speed. This paper investigates the uncertainty analysis for anemometer calibration testing as defined in the IEC standard, in which anemometer calibration uncertainty in only defined through the propagation of errors in the reference wind speed. Nonetheless, uncertainty in the reference wind speed is an appropriate baseline to the total uncertainty in the calibration. Thus, this paper also proposes an expanded uncertainty analysis, which would incorporate additional sources of error, including the uncertainty in the output of the anemometer and particularly in the use of the linear transfer equation calculated from the anemometer calibration. As common practice, the calibration transfer function is used to convert the anemometer output measured in the field to the corresponding wind speed.

B. Anemometer Calibration Standards

Anemometer calibration is performed to determine the relationship between the output of the anemometer, whether a voltage or TTL signal (i.e., Hz or RPM), and the measurement of the reference wind speed. Thus, during a calibration test anemometer output is collected for a range of wind speed settings. With this data a regression analysis is conducted to determine a calibration transfer function. Ideally, this relationship is linear (see Figure 2). The wind speed residuals can also be determined by finding the difference between the calculated wind speed based on the linear regression equation and the measured reference speed. A sample wind speed residual graph is presented in Figure 3. From the linear regression analysis, two statistical measures are also calculated and used to represent the degree of linearity between the anemometer output and corresponding wind speed readings.

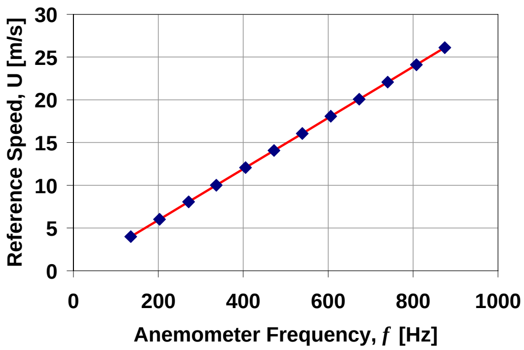


Figure 2: Sample calibration result with linear regression line.

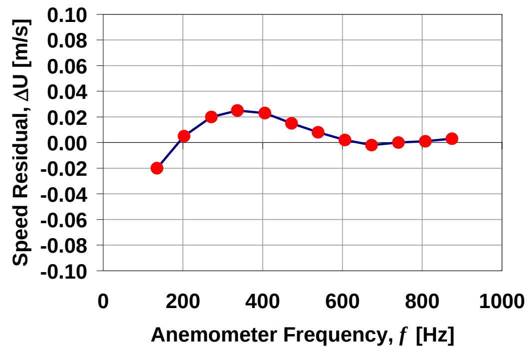


Figure 3: Sample wind speed residual graph of an anemometer calibration.

Several techniques of anemometer calibration have been attempted including using a moving vehicle where the anemometer under test is moved through the air. A more controlled test methodology is done using a wind tunnel where air is moved across the anemometer under test. Some facilities test one anemometer at a time while there are also those that test multiple anemometers during one calibration cycle. In order to reduce the biases, it is recommended that anemometer calibration test standards be complied. In the wind energy industry, the most commonly referred publication for anemometer calibration is IEC 61400-12-1, released in December 2005. This particular document provides calibration procedures for cup anemometers used in turbine power performance evaluation. A calibration protocol for both cup and propeller anemometers is also provided in ASTM D 5096-02, originally published in 1990. This particular standard applies to anemometers used for general meteorology applications including wind resource assessment. In May 2007, ISO 17713-1 was released, which is an international standard for calibration and performance testing of rotating anemometers. This ISO standard is essentially an updated version of ASTM D 5096-02 and refers to ISO 5725-1 and ISO 5725-2 as a guideline for calculating measurement uncertainty. Although some of the details in these standards differ in some ways, a common requirement for anemometer calibration and performance evaluation is that tests are to be conducted under controlled conditions using a low turbulence, uniform-flow wind tunnel.

C. Introduction to Uncertainty

From its basic definition, uncertainty is an estimate of the errors in a measured variable. It defines the propagation of bias, β, and precision, σ, errors that surround a particular measurement as shown in Figure 4. Here, X_{true} is the true value for a particular variable, such as wind speed, and $\overline{X}$ is a measured variable from a certain instrument system, such as a wind tunnel Pitot-static tube system. The bias error, β, is the fixed error that defines the offset of $\overline{X}$ from X_{true}. Some references identify the bias as systematic errors or Type B errors according to NIST. With multiple readings of $\overline{X}$, precision errors, based on the variability of the readings at a particular statistical confidence interval, define repeatability or variability of $\overline{X}$. Precision errors are also known as random errors or Type A errors according to NIST.

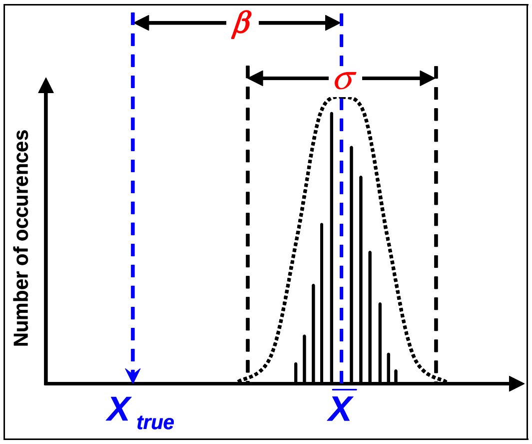


Figure 4: Bias and random error contributions to uncertainty, β and σ respectively.

Uncertainty is of great importance and can be a powerful tool in the wind energy industry such that it could be used to define whether or not a wind farm project is successful. For developers and finance institutions, the feasibility of a project and the degree of financial risks and returns are predicted partly by wind power uncertainty estimates. For wind energy consultants, an uncertainty analysis can be used to track down sources that generate errors in a wind energy estimate so that recommendations may be made to improve the system. Unfortunately, uncertainty can also be made misleading when presented based on incomplete analysis in error propagation or for incorrect use of terminology. At times uncertainty is referred to as a measure of accuracy when, on the contrary, accuracy is a form of bias error, β, that is a partial role in uncertainty. By definition, accuracy is a closeness of agreement between a measured, $\overline{X}$, and a true value, X_{true}.

D. IEC 61400-12-1 Uncertainty Analysis Method

There are several references providing methods in determining the uncertainty of an anemometer calibration. For the wind energy industry, the most widely referred standard is IEC 61400-12-1, first edition, 2005-12: "Power Performance Measurements of Electricity Producing Wind Turbines". This particular document provides the steps in conducting a field evaluation of a wind turbine producing a power curve. The evaluation includes site measurements of the local wind speed. Within the appendices of this standard are procedures in performing a cup anemometer calibration transfer function test along with the various tests that would evaluate the instrument's sensitivity to certain terrain and atmospheric conditions. The standard also specifies that one should conduct cup anemometer calibration using a wind tunnel test facility that incorporates a Pitot-static tube system to measure the reference wind speed. As a note, some test facilities choose to incorporate other methods to measure the reference speed such as hot wire anemometry, laser Doppler velocimetry, or even propeller or other types of dynamic anemometers. In the IEC standard, however, uncertainty analysis is conducted based on the errors accumulated in the reference wind speed measurement from a Pitot-static tube system.

In a test protocol where an anemometer is calibrated to the wind speed measurement sensed by the Pitot-static tube system, the IEC standard suggests that the uncertainty in the calibration is defined by the uncertainty in the reference wind speed, V, defined in Equation (1).

$$V = k_b \sqrt{\frac{2k_c}{C_h}\frac{\Delta p}{\rho}} \qquad \text{Eq. (1)}$$

Here, Δp is the differential pressure reading from the Pitot-static tube, C_h is the Pitot-static tube head coefficient, ρ is the density, k_c is the wind tunnel calibration factor, and k_b is the blockage correction. In more detail, the density can be defined in terms of the ambient pressure, P, the ambient temperature, T, the relative humidity, ϕ, vapor pressure, P_w, the gas constant for air, R_{air}, and the gas constant for water, R_w, as defined in Equation (2).

$$\rho = \frac{1}{T}\left[\frac{P}{R_{air}} - 0.01\phi P_w\left(\frac{1}{R_{air}} - \frac{1}{R_w}\right)\right] \qquad \text{Eq. (2)}$$

Vapor pressure and gas constants for air and water are then defined as follows:

$$P_w = 0.0000205\exp(0.0631846T) \qquad \text{Eq. (3)}$$

$$R_{air} = \frac{R}{M_{air}} \qquad \text{Eq. (4)}$$

$$R_w = \frac{R}{M_w} \qquad \text{Eq. (5)}$$

Here, R is the universal gas constant. M_{air} and M_w are the molecular weights for air and water, respectively. Using the terms listed above, the reference wind speed measured from a Pitot-static tube system previously defined in Equation (1) can be expanded as follows.

$$V = k_b \sqrt{\frac{2k_c \Delta p R T}{C_h\left[PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air} - M_w)\right]}} \qquad \text{Eq. (6)}$$

This form of the wind speed equation allows a more direct identification of the variables that are most sensitive to the calculation. From Equation (6), the independent variables, terms with exact values as defined in NIST, are found to be M_{air} and M_w. Since independent variables have no systematic or random errors, further sensitivity analysis are not required. Dependent variables are essentially measured or pre-calculated parameters which do require further analysis. Thus, from Equation (6), uncertainty in the reference wind speed measured by a Pitot-static tube system is a function of k_b, k_c, C_h, R, P, T, Δp, and ϕ and is then defined as:

$$U_V = \sqrt{(B_V)^2 + (tS_V)^2} \qquad \text{Eq. (7)}$$

Here, B_V represents the propagation of systematic or bias error contributions and is a function of all the dependent variables found in Equation (6). Thus, the propagation of systematic errors is defined as:

$$B_V = \sqrt{\begin{array}{c}\left(\frac{\delta V}{\delta k_b}B_{k_b}\right)^2 + \left(\frac{\delta V}{\delta k_c}B_{k_c}\right)^2 + \left(\frac{\delta V}{\delta C_h}B_{C_h}\right)^2 + \left(\frac{\delta V}{\delta R}B_R\right)^2 \\ + \left(\frac{\delta V}{\delta P}B_P\right)^2 + \left(\frac{\delta V}{\delta T}B_T\right)^2 + \left(\frac{\delta V}{\delta \Delta p}B_{\Delta p}\right)^2 + \left(\frac{\delta V}{\delta \phi}B_\phi\right)^2\end{array}} \qquad \text{Eq. (8)}$$

In Equation (8) above, B_{k_b}, B_{k_c}, B_{C_h}, B_R, B_P, B_T, $B_{\Delta p}$, and B_ϕ are the bias errors from each of the dependent variables. For the measured variables, P, T, Δp, and ϕ, bias errors can be found from data acquisition, signal conditioning, and instrument performance such as linearity or accuracy. For the assigned or property variables, k_b, k_c, C_h, and R, "fossilized" errors are generally applied, representing both the random and systematic errors in the determination of such variables.

From Equation (7), S_V signifies the propagation of random or precision error contributions which originate only from the measured dependent variables, P, T, Δp, and ϕ. The value of t for 95% confidence at ∞ degrees of freedom is 1.96[4]. The propagation of random errors is defined as according to the following equation.

$$S_V = \sqrt{\left(\frac{\delta V}{\delta P} S_P\right)^2 + \left(\frac{\delta V}{\delta T} S_T\right)^2 + \left(\frac{\delta V}{\delta \Delta p} S_{\Delta p}\right)^2 + \left(\frac{\delta V}{\delta \phi} S_\phi\right)^2} \qquad \text{Eq. (9)}$$

Random errors are the variability in the measured variables. Thus, S_P, S_T, $S_{\Delta p}$, and S_ϕ, are simply the standard deviations of the mean values from the corresponding measured variables. For both Equations (8) and (9), the partial differentials in front of each term are the corresponding sensitivity coefficients of each dependent variable. These partial differentials are essentially derived from the expanded reference wind speed, Equation (6). Below displays a table of the sensitivity equations required in Equation (6).

Table 1: Sensitivity coefficient partial differential equations for each dependent variable.

Dependent Variable	Sensitivity Coefficient Equation	Equation Number
Blockage Coefficient	$\frac{\delta V}{\delta k_b} = \sqrt{\frac{2\Delta pRT}{C_h\left[PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T_K)(M_{air}-M_w)\right]}} = \frac{V}{k_b}$	Eq. (10)
Pitot-static Tube Head Coefficient	$\frac{\delta V}{\delta C_h} = -\frac{1}{2}\frac{k_b}{C_h^{3/2}}\sqrt{\frac{2\Delta pRT}{PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)}} = -\frac{1}{2}\frac{V}{C_h}$	Eq. (11)
Universal Gas Constant	$\frac{\delta V}{\delta R} = \frac{1}{2}k_b\sqrt{\frac{2\Delta pT}{RC_h\left[PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]}} = \frac{1}{2}\frac{V}{R}$	Eq. (12)
Ambient Pressure	$\frac{\delta V}{\delta P} = -\frac{1}{2}\frac{k_b M_{air}}{\left[PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]^{3/2}}\sqrt{\frac{2\Delta pRT}{C_h}} = -\frac{1}{2}\frac{VM_{air}}{PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)}$	Eq. (13)
Ambient Temperature	$\frac{\delta V}{\delta T_K} = k_b\sqrt{\frac{2\Delta pR}{C_h}}\frac{0.5(T_K)^{-1/2}\left[PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]^{1/2} - 0.5\left[PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]^{-1/2}(T)^{1/2}\left(-0.0631846\left[2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]\right)}{PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)} = \frac{1}{2}V\left[\frac{1}{T} + \frac{1.295\times10^{-8}\phi\exp(0.0631846T)(M_{air}-M_w)}{PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)}\right]$	Eq. (14)
Differential Pressure	$\frac{\delta V}{\delta \Delta p} = \frac{1}{2}k_b\sqrt{\frac{2RT}{\Delta pC_h\left[PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]}} = \frac{1}{2}\frac{V}{\Delta p}$	Eq. (15)
Relative Humidity	$\frac{\delta V}{\delta \phi} = -\frac{1}{2}\frac{k_b\left[-2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]}{\left[PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]^{3/2}}\sqrt{\frac{2\Delta pRT}{C_h}} = \frac{1}{2}\frac{V\left[-2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)\right]}{PM_{air} - 2.05\times10^{-7}\phi\exp(0.0631846T)(M_{air}-M_w)}$	Eq. (16)

An uncertainty analysis using the IEC 61400-12-1 methodology was conducted for the Otech Engineering Wind Tunnel Facility. This facility is a uniform-flow, low-turbulence wind tunnel which uses a Pitot-static tube system to measure the reference wind speed (see Figure 5 and Figure 6). A sample calibration report is presented in Figure 7.

Figure 5: Otech Engineering Wind Tunnel Facility located at Davis, CA.

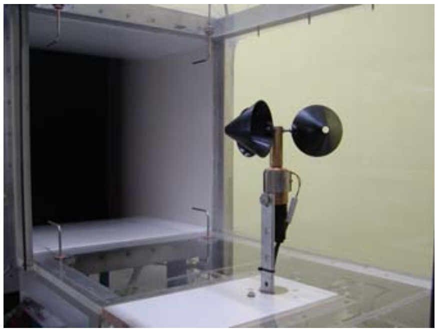
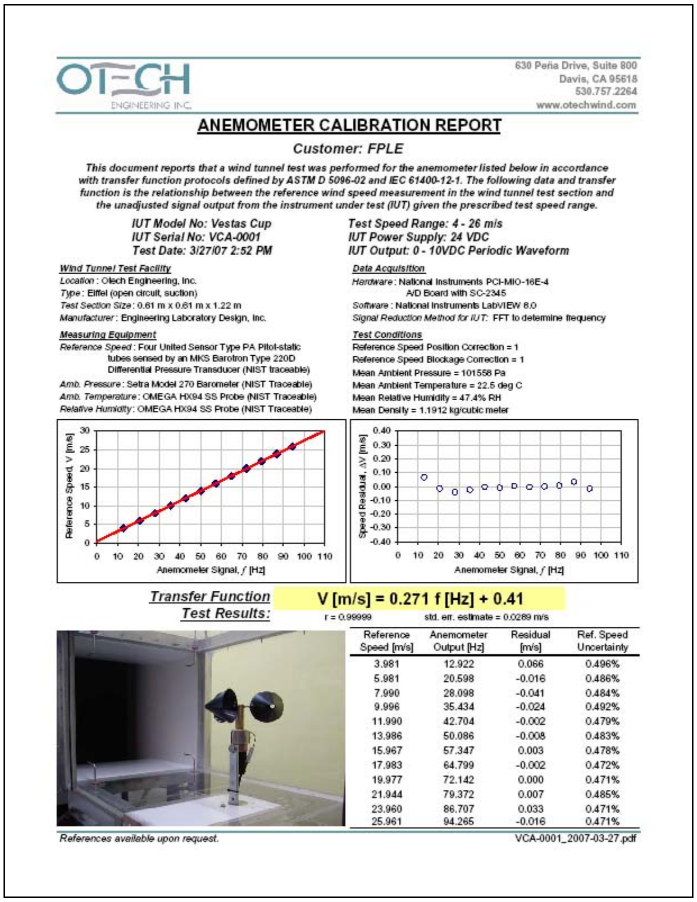


Figure 6: Anemometer calibration testing using a Pitot-static tube system.



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ANEMOMETER CALIBRATION REPORT

Customer: FPLE

This document reports that a wind tunnel test was performed for the anemometer listed below in accordance with transfer function protocols defined by ASTM D 5096-02 and IEC 61400-12-1. The following data and transfer function is the relationship between the reference wind speed measurement in the wind tunnel test section and the unadjusted signal output from the instrument under test (IUT) given the prescribed test speed range.

IUT Model No: Vestas Cup
IUT Serial No: VCA-0001
Test Date: 3/27/07 2:52 PM

Test Speed Range: 4 - 26 m/s
IUT Power Supply: 24 VDC
IUT Output: 0 - 10VDC Periodic Waveform

Wind Tunnel Test Facility
Location: Otech Engineering, Inc.
Type: Eiffel (open circuit, suction)
Test Section Size: 0.61 m x 0.61 m x 1.22 m
Manufacturer: Engineering Laboratory Design, Inc.

Measuring Equipment
Reference Speed: Four United Sensor Type PA Pitot-static tubes sensed by an MKS Barotron Type 220D Differential Pressure Transducer (NIST traceable)
Amb. Pressure: Setra Model 270 Barometer (NIST Traceable)
Amb. Temperature: OMEGA HX94 SS Probe (NIST Traceable)
Relative Humidity: OMEGA HX94 SS Probe (NIST Traceable)

Data Acquisition
Hardware: National Instruments PCI-MIO-16E-4 A/D Board with SC-2345
Software: National Instruments LabVIEW 8.0
Signal Reduction Method for IUT: FFT to determine frequency

Test Conditions
Reference Speed Position Correction = 1
Reference Speed Blockage Correction = 1
Mean Ambient Pressure = 101558 Pa
Mean Ambient Temperature = 22.5 deg C
Mean Relative Humidity = 47.4% RH
Mean Density = 1.1912 kg/cubic meter

Transfer Function **V [m/s] = 0.271 f [Hz] + 0.41**

Test Results: r = 0.99999 std. err. estimate = 0.0289 m/s

Reference Speed [m/s]	Anemometer Output [Hz]	Residual [m/s]	Ref. Speed Uncertainty
3.981	12.922	0.066	0.496%
5.981	20.598	-0.016	0.486%
7.990	28.098	-0.041	0.484%
9.996	35.434	-0.024	0.492%
11.990	42.704	-0.002	0.479%
13.986	50.086	-0.008	0.483%
15.967	57.347	0.003	0.478%
17.983	64.799	-0.002	0.472%
19.977	72.142	0.000	0.471%
21.944	79.372	0.007	0.485%
23.960	86.707	0.033	0.471%
25.961	94.265	-0.016	0.471%

References available upon request. VCA-0001_2007-03-27.pdf

Figure 7: Sample anemometer calibration report generated at Otech Engineering.

In the sample calibration report above, specifications of both the anemometer under test and the wind tunnel facility are first presented, including a list of the instruments used to measure the wind tunnel speed and corresponding local conditions (i.e., ambient pressure, temperature, and relative humidity). Results from the test are also reported, which includes the measured local conditions. From the table of test speeds and corresponding anemometer output measurements, a column of the corresponding uncertainty for each wind speed is also presented. According to IEC 61400-12-1 guidelines, the average uncertainty in the reference wind speed measurement in the Otech Wind Tunnel Facility for test speeds ranging from 4 to 26 m/s is approximately 0.5%.

E. Expanded Uncertainty Analysis for Anemometer Calibration

In the previous section, a current method of uncertainty presentation of an anemometer calibration was conducted using the uncertainty analysis in the reference wind speed measurement as defined in IEC 61400-12-1. However, this uncertainty may only apply if you use the calibration tables as a "look-up" table to convert the anemometer signal output into wind speed. In addition, since anemometer calibration relies on the output of the anemometer itself, an uncertainty analysis of the anemometer signal must also be investigated. If the linear regression equation of the calibration tables is used, the uncertainty in the regression should also be accounted for in the anemometer calibration. Thus, an expanded uncertainty of the anemometer calibration, U_{cal}, would be the sum of the squares of the uncertainty in the reference wind speed measurement as defined in IEC 61400-12-1, U_V, the uncertainty in the output of the anemometer under test, U_{IUT}, and the uncertainty in the linear regression fit, U_{LR}.

$$U_{cal} = \sqrt{(U_V)^2 + (U_{IUT})^2 + (U_{LR})^2} \qquad \text{Eq. (17)}$$

Uncertainty in the output of the test anemometer, U_{IUT}, is essentially the propagation of inherent bias errors, B_{IUT}, and precision errors generated during the calibration test, S_{IUT} (see Equation 18). Bias errors in the anemometer output are primarily sourced from the data acquisition system and, for the pulse output anemometer, the methodology of calculating for rate of rotation whether in Hz or rpm. The precision errors are accounted for in the standard deviation of anemometer output reading during the duration of the data collection. Again, the value of t for 95% confidence at ∞ degrees of freedom is 1.96[4]. Bias errors are typically dominated by the method of determining the anemometer's rate of rotation. For instance, with a pulse count method, a bias error is associated with the least significant count. Precision errors may vary largely based on the standard deviations in the anemometer output at each wind speed. Thus, typically the range in the anemometer output uncertainty could vary from 0.5% to 1.5%.

$$U_{IUT} = \sqrt{B_{IUT}^2 + (tS_{IUT})^2} \qquad \text{Eq. (18)}$$

Linear regression uncertainty, U_{LR}, may be approached in one of two options. First is the classical method, which then can also be done in one of two ways. One is by applying the standard error of estimate in the linear fit (STE_V) as shown in Equation 19, where STE_V is defined in Equation 20.

$$U_{LR} = \sum_{i=1}^{N} t \frac{STE_V}{V_i} \qquad \text{Eq. (19)}$$

$$STE_V = \sqrt{\frac{\sum_{i=1}^{N} (V_i - mf_i - b)^2}{N-2}} \qquad \text{Eq. (20)}$$

Defining the uncertainty in the linear regression using Equation 19 is more applicable from most anemometer calibration reports which report the STE_V. However, if the standard error in the slope (STE_m) and the offset (STE_b) are calculated and readily available, then the uncertainty in the linear regression analysis would be based on the propagation of such errors as shown in Equation 21, where STE_m and STE_b are defined in equations 22 and 23.

$$U_{LR} = \sqrt{\left(t \frac{STE_m}{V_i} f_i\right)^2 + \left(t \frac{STE_b}{V_i}\right)^2} \qquad \text{Eq. (21)}$$

$$STE_m = \sqrt{\frac{STE_V^2}{SS_f}} \qquad \text{Eq. (22)}$$

$$STE_b = \sqrt{STE_V^2 \left(\frac{1}{N} + \frac{\bar{f}^2}{SS_f}\right)} \qquad \text{Eq. (23)}$$

Here, the term SS_f is the sum of the squares of the anemometer readings measured at each test wind speed. This is calculated according to the following Equation 24.

$$SS_f = \sum_{i=1}^{N} f_i^2 - \frac{\left(\sum_{i=1}^{N} f_i\right)}{N} \qquad \text{Eq. (24)}$$

At Otech, values of the STE_V for various types of anemometers could in range from 0.02 m/s to as much as 0.15 m/s for a test range of 4 to 26 m/s. Thus, using Equation 19, the linear regression uncertainty could vary from 0.2% to 5.0% depending on the STE_V. For the most part, STE_m and STE_b are smaller compared to STE_V, however, when combined in the propagation of uncertainty as defined in Equation 21, the linear regression uncertainty can vary from 0.2% to 5.0% similar to the results from Equation 19. For the second method, the dominant error source is generally the standard error in the offset.

Using Equation 19 or 21 requires that all the random errors in determining the linear transfer function only originate from the wind speed reading; however, in reality, during a calibration the reading from the anemometer output is also measured. Thus, random errors in the anemometer output should also be accounted for. A second option in defining the uncertainty in the linear regression involves a more comprehensive expression for U_{LR}, which assumes that random errors in both the reference wind speed and the anemometer output dominate the analysis. For this case, U_{LR} is defined as follows.

$$U_{LR} = \sqrt{U_m^2 + U_b^2} \qquad \text{Eq. (23)}$$

Here, U_m and U_b are the uncertainty values for the slope, m, and offset, b, respectively. In a linear regression analysis, the slope and offset are a function of the two variables to be related. For the case of anemometer calibration, the two variables are wind speed and anemometer output. Below are the equations for the slope and offset in an anemometer calibration linear regression analysis.

$$m = \frac{N\sum_{i=1}^{N} f_i V_i - \sum_{i=1}^{N} f_i \sum_{i=1}^{N}\left(f_i V_i\right)}{N\sum_{i=1}^{N}\left(f_i^2\right) - \left(\sum_{i=1}^{N} f_i\right)^2} \qquad \text{Eq. (24)}$$

$$b = \frac{\sum_{i=1}^{N}\left(f_i^2\right)\sum_{i=1}^{N} V_i - \sum_{i=1}^{N} f_i \sum_{i=1}^{N}\left(f_i V_i\right)}{N\sum_{i=1}^{N}\left(f_i^2\right) - \left(\sum_{i=1}^{N} f_i\right)^2} \qquad \text{Eq. (25)}$$

As shown in Equations 24 and 25, the slope and offset are both a function of the reference wind speed, V_i, and the anemometer output, f_i. Thus, the general comprehensive expressions for the uncertainty in the slope and offset are as follows, which includes the bias error propagation of the correlated variables.

$$\begin{aligned} U_m^2 = {} & \sum_{i=1}^{N}\left(\frac{\delta m}{\delta V_i}\right)^2 S_{V_i}^2 + \sum_{i=1}^{N}\left(\frac{\delta m}{\delta f_i}\right)^2 S_{f_i}^2 + \sum_{i=1}^{N}\left(\frac{\delta m}{\delta V_i}\right)^2 B_{V_i}^2 + 2\sum_{i=1}^{N-1}\sum_{k=i+1}^{N}\left(\frac{\delta m}{\delta V_i}\right)^2\left(\frac{\delta m}{\delta V_k}\right)^2 B_{V_i V_k} \\ & + \sum_{i=1}^{N}\left(\frac{\delta m}{\delta f_i}\right)^2 B_{f_i}^2 + 2\sum_{i=1}^{N-1}\sum_{k=i+1}^{N}\left(\frac{\delta m}{\delta f_i}\right)^2\left(\frac{\delta m}{\delta f_k}\right)^2 B_{f_i f_k} + 2\sum_{i=1}^{N-1}\sum_{k=1}^{N}\left(\frac{\delta m}{\delta f_i}\right)^2\left(\frac{\delta m}{\delta V_k}\right)^2 B_{f_i V_k} \end{aligned} \qquad \text{Eq. (26)}$$

$$U_b^2 = \sum_{i=1}^{N}\left(\frac{\delta b}{\delta V_i}\right)^2 S_{V_i}^2 + \sum_{i=1}^{N}\left(\frac{\delta b}{\delta f_i}\right)^2 S_{f_i}^2 + \sum_{i=1}^{N}\left(\frac{\delta b}{\delta V_i}\right)^2 B_{V_i}^2 + 2\sum_{i=1}^{N-1}\sum_{k=i+1}^{N}\left(\frac{\delta b}{\delta V_i}\right)^2\left(\frac{\delta b}{\delta V_k}\right)^2 B_{V_iV_k}$$
$$+\sum_{i=1}^{N}\left(\frac{\delta b}{\delta f_i}\right)^2 B_{f_i}^2 \quad +2\sum_{i=1}^{N-1}\sum_{k=i+1}^{N}\left(\frac{\delta b}{\delta f_i}\right)^2\left(\frac{\delta b}{\delta f_k}\right)^2 B_{f_i f_k} + 2\sum_{i=1}^{N-1}\sum_{k=1}^{N}\left(\frac{\delta b}{\delta f_i}\right)^2\left(\frac{\delta b}{\delta V_k}\right)^2 B_{f_iV_k} \qquad \text{Eq. (27)}$$

In Equations 26 and 27, B_{V_i} and B_{f_i} are the propagation of the corresponding bias errors, respectively. The fourth and sixth terms in Equations 26 and 27 accounts for the self-correlated bias errors for both the wind speed and anemometer output measurements for a range of *N* test speeds. The last term in both Equations 26 and 27 defines the correlated bias errors between the wind speed and the anemometer output readings at each test speed. If there are no common sources of error such as a similar data acquisition system, then the last term goes to zero. The variables S_{V_i} and S_{f_i} are the propagation of precision errors in the wind speed and the anemometer output, respectively. For each test speed, a standard deviation in the wind speed and anemometer reading is determined for data collected during a particular acquisition time. This standard deviation at 95% confidence is generally used to determine the precision of the measurement.

More investigation is required in the comprehensive analysis for the linear regression uncertainty, particularly in the sensitivity coefficients. Further research would determine whether a comprehensive analysis would be required or whether using the classical expression would provide a valid representation for linear regression fit uncertainty. Another key note would be that if the calibration results were used as a “look up” table to determine the wind speed reading for a particular anemometer output in the field, the uncertainty in the anemometer calibration would be directly related only to the uncertainty in the reference wind speed and in the output of the anemometer in the test facility. However, the most common practice is to use the calibration equation. In this case, the uncertainty should also reflect the uncertainty contribution from the linear regression fit. Table 1 is a summary of the equations required for an expanded uncertainty analysis using the classical expression for linear regression uncertainty. Since the standard error in the linear regression is most commonly provided in calibration reports, Case 1 in the table below would provide an applicable expression for linear regression uncertainty.

Table 2: Summary of equations for anemometer calibration expanded uncertainty analysis.

Anemometer Calibration Uncertainty	$U_{cal} = \sqrt{(U_V)^2 + (U_{IUT})^2 + (U_{LR})^2}$	~ 0.7% to 5.2%
Reference Wind Speed Uncertainty	$U_V = \sqrt{(B_V)^2 + (tS_V)^2}$	~ 0.5%
Anemometer Output Uncertainty	$U_{IUT} = \sqrt{B_{IUT}^2 + (tS_{IUT})^2}$	~ 0.5% to 1.5%
Linear Regression Uncertainty (Case 1)	$U_{LR} = \sum_{i=1}^{N} t\frac{STE_V}{V_i}$	~ 0.2% to 5.0 %
Linear Regression Uncertainty (Case 2)	$U_{LR} = \sqrt{\left(t\frac{STE_m}{V_i} f_i\right)^2 + \left(t\frac{STE_b}{V_i}\right)^2}$	~ 0.2% to 5.0 %

An expanded uncertainty analysis was conducted using the data presented in the sample calibration report shown in Figure 7. For this analysis, the linear regression uncertainty was calculated based on the presentation of the standard error of estimate in the linear fit. From this sample calibration case, the standard error of estimate was 0.0289 m/s revealing a highly linear sensor. A lower standard error value generally corresponds to a more linear anemometer. The bias uncertainty in the anemometer output measurement was essentially based on the errors in the data acquisition system. In the following Table 3, the lower right-hand results table portion from the calibration report in Figure 7 is presented along with columns of the calculated values of anemometer output uncertainty, regression uncertainty, and the calibration uncertainty for each test speed. From this sample analysis, the average anemometer calibration uncertainty for a test speed range of 4 to 26 m/s was found to be 1.2%.

Table 3: Expanded uncertainty analysis for sample calibration given in Figure 7.

Reference Speed [m/s]	Anemometer Output [Hz]	Residual [m/s]	Ref. Speed Uncertainty, U_V	Anem. Output Uncertainty, U_{IUT}	Regression Uncertainty, U_{LR}	Calibration Uncertainty, U_{cal}
3.981	12.922	0.066	0.496%	1.467%	1.425%	2.104%
5.981	20.598	-0.016	0.486%	1.030%	0.949%	1.482%
7.990	28.098	-0.041	0.484%	0.981%	0.710%	1.304%
9.996	35.434	-0.024	0.492%	0.780%	0.567%	1.083%
11.990	42.704	-0.002	0.479%	0.818%	0.473%	1.059%
13.986	50.086	-0.008	0.483%	0.954%	0.406%	1.144%
15.967	57.347	0.003	0.478%	0.715%	0.355%	0.930%
17.983	64.799	-0.002	0.472%	0.785%	0.315%	0.969%
19.977	72.142	0.000	0.471%	0.802%	0.284%	0.973%
21.944	79.372	0.007	0.485%	0.938%	0.259%	1.087%
23.960	86.707	0.033	0.471%	1.071%	0.237%	1.194%
25.961	94.265	-0.016	0.471%	1.086%	0.219%	1.204%
Average Uncertainties			**0.5%**	**1.0%**	**0.5%**	**1.2%**

F. Application of Anemometer Calibration Uncertainty

Contributions to the uncertainty in wind energy estimates may originate from several sources of error such as wind variability at a selected site, resource modeling, or field measurements of wind. Field measurements are essentially conducted using anemometer towers. Typically, the most common protocol in field anemometry is to program a calibration transfer function into data loggers or other form of data acquisition system to convert the output signal from an anemometer into a wind speed reading. Thus, the equation used to conduct uncertainty analysis in a field measured wind speed is as follows:

$$V_{field} = mf_{field} + b \qquad \text{Eq. (14)}$$

Here, V_{field} is the measured wind speed at a particular site, f_{field} is the anemometer output at the site, m and b are the slope and offset, respectively, from the linear regression of the anemometer calibration. Based on this equation, the uncertainty in the field measured wind speed is a function of m, b, and f_{field}. Assuming that the anemometer readings during its calibration have no random errors, the classical uncertainty expression in the anemometer calibration may be used to define the combined uncertainty in the use of the calibration slope and offset. Thus, the uncertainty in the field measured wind speed is a function of the uncertainty in the anemometer calibration, U_{cal}, as defined in Equation 17, and uncertainty in the field measured output from the anemometer, $U_{f_{field}}$.

$$U_{V_{field}} = \sqrt{(U_{cal})^2 + (U_{f_{field}})^2} \qquad \text{Eq. (15)}$$

In the previous section, the anemometer calibration uncertainty was quantified based on an expanded uncertainty analysis method. To determine the uncertainty of the wind speed measured in the field, a review of the sources of uncertainty in the field measured output of the anemometer is necessary. In a qualitative perspective, the uncertainty in the anemometer field output is essentially a function of the installation (i.e., tower installation, data acquisition system, etc.) and its sensitivities to the multiple types of environmental conditions (i.e., temperature, terrain complexities, atmospheric turbulence and stability, etc.). This generally emphasizes that anemometer calibration is only a portion of the sources of uncertainty in the field measured wind speed.

G. Conclusion

Under current standards (IEC 61400-12-1), anemometer calibration uncertainty is based only on the uncertainty of the reference wind speed measured at the test facility. For field applications, a more useful value would be an expanded uncertainty analysis of the anemometer calibration which not only includes the test facility wind speed uncertainty but also the uncertainty in the anemometer output measured in the test facility and, if applied, the uncertainty in the use of the calibration linear transfer function equation. The uncertainty in the linear transfer function would not be required if the calibration results were applied as a “look up” table. Common field practice uses the linear regression equation. This paper presented versions of the classical and comprehensive approach in the uncertainty analysis of the linear regression equation. The comprehensive approach required further investigation in the sensitivity coefficients to determine whether it would be necessary to use a comprehensive uncertainty analysis or just to apply the classical approach. However, it was also determined that the classical approach does provide a valid estimate of the uncertainty in the linear regression fit. Thus, the expanded anemometer calibration uncertainty is the propagation of errors from the reference wind speed measurement at the test facility and includes contributions from the linear regression. When applied to the field, the anemometer calibration is only one of many sources of uncertainty in the field measured wind speed. For this case, the uncertainty in the anemometer output due to installation and to sensitivities to environmental conditions should be accounted for.

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